

Session 3

Basics of Vibration Theory and Application

Vibration In Everyday Life



Sources of Vibration

- All bodies possessing mass and elasticity are capable of vibration.
- Most engineering machines and structures are capable of vibration.
- The design of such machines and structures generally requires consideration of their oscillatory behavior.

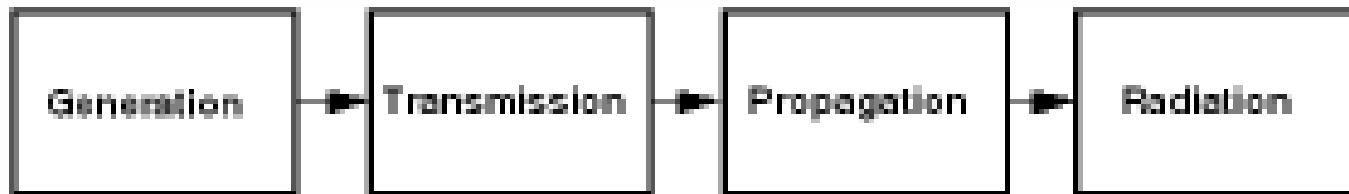
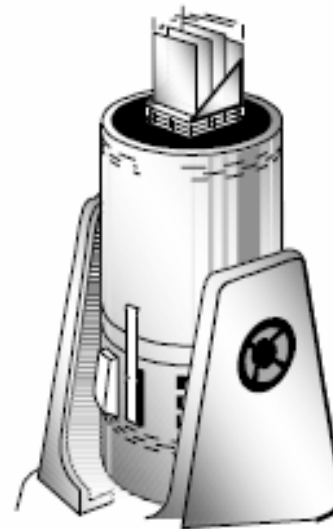
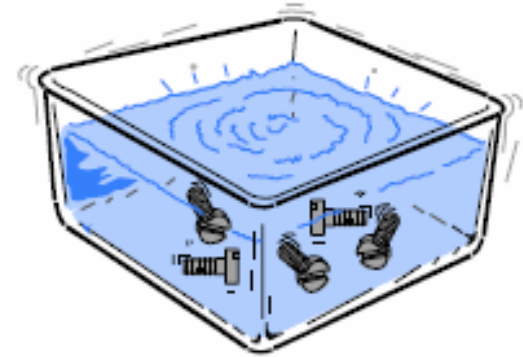


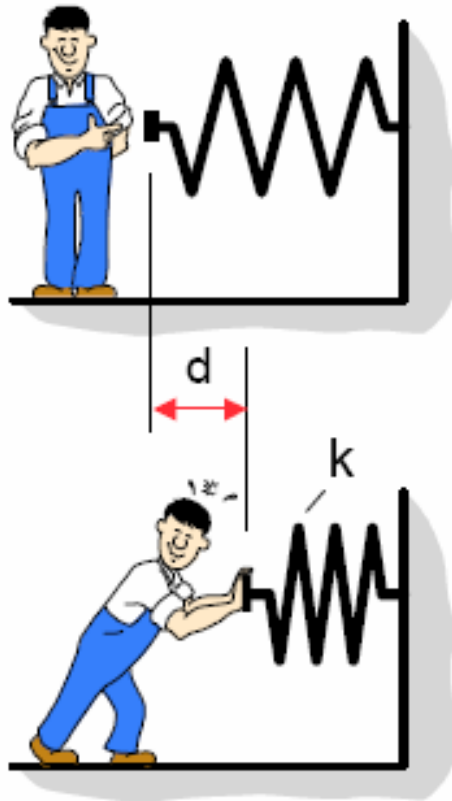
Figure 1 Mechanical vibration as a process.

Useful Vibration



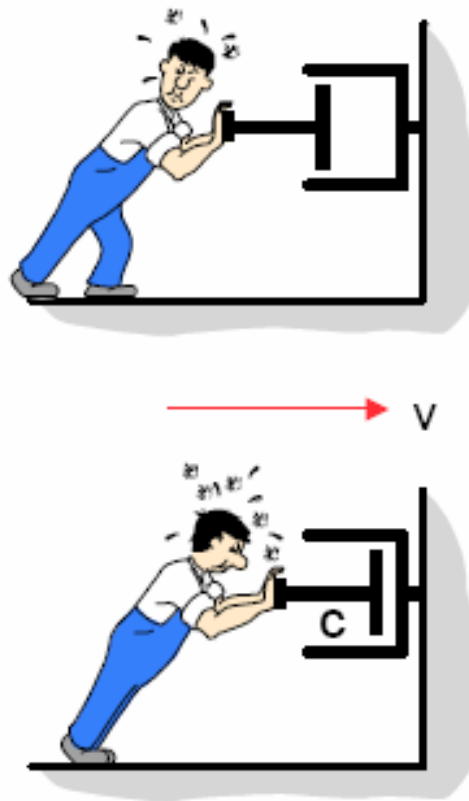
Mechanical Parameters and Components

Displacement



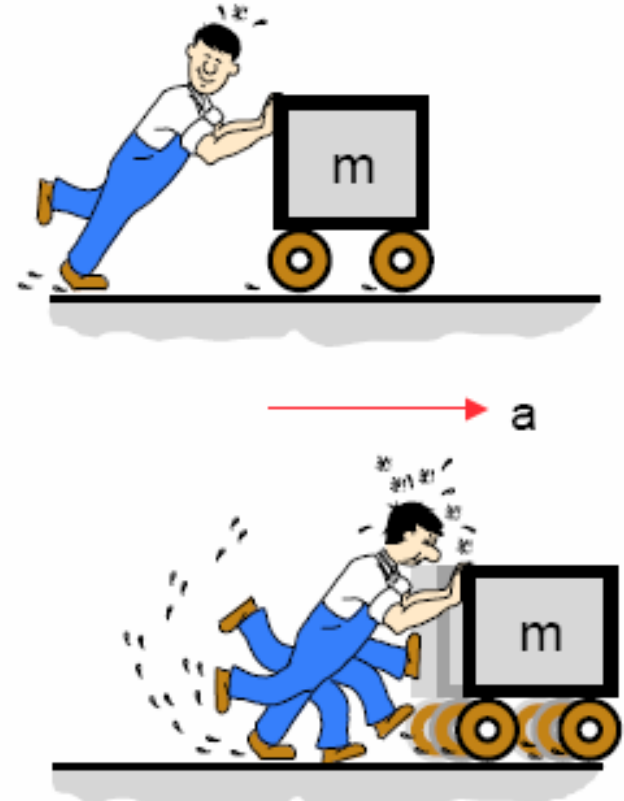
$$F = k \times d$$

Velocity



$$F = c \times v$$

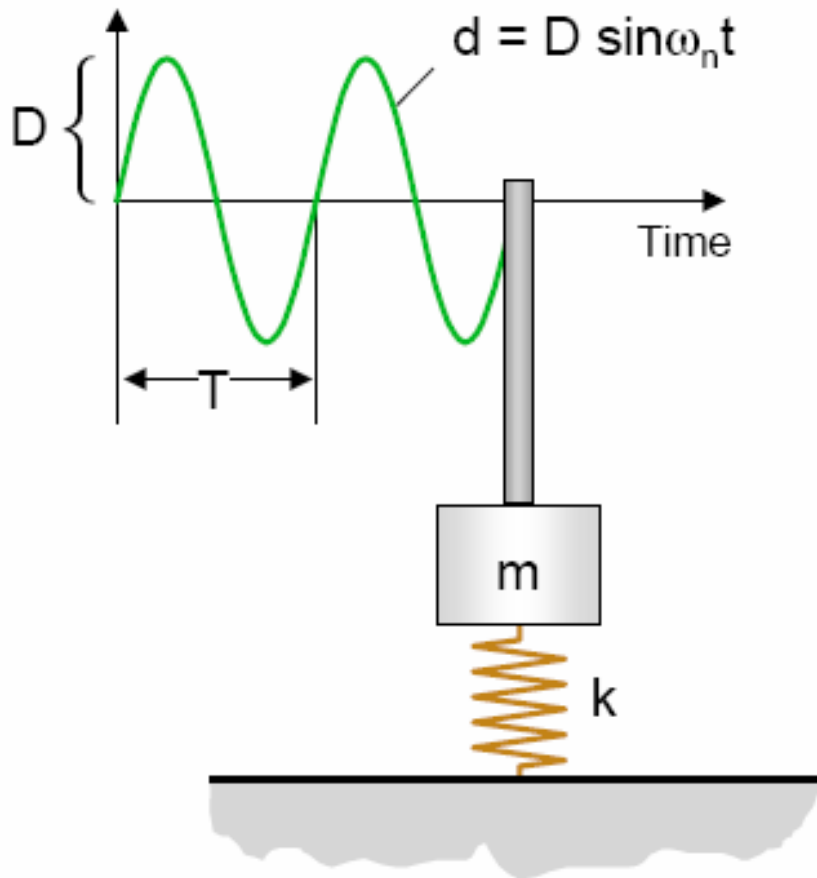
Acceleration



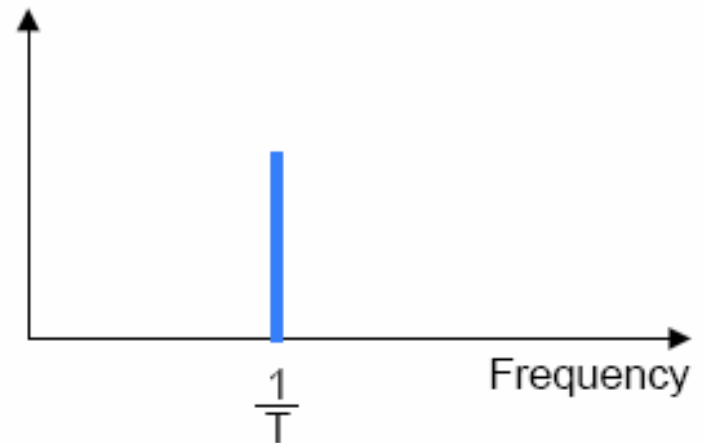
$$F = m \times a$$

Simplest Form of Vibrating System

Displacement



Displacement

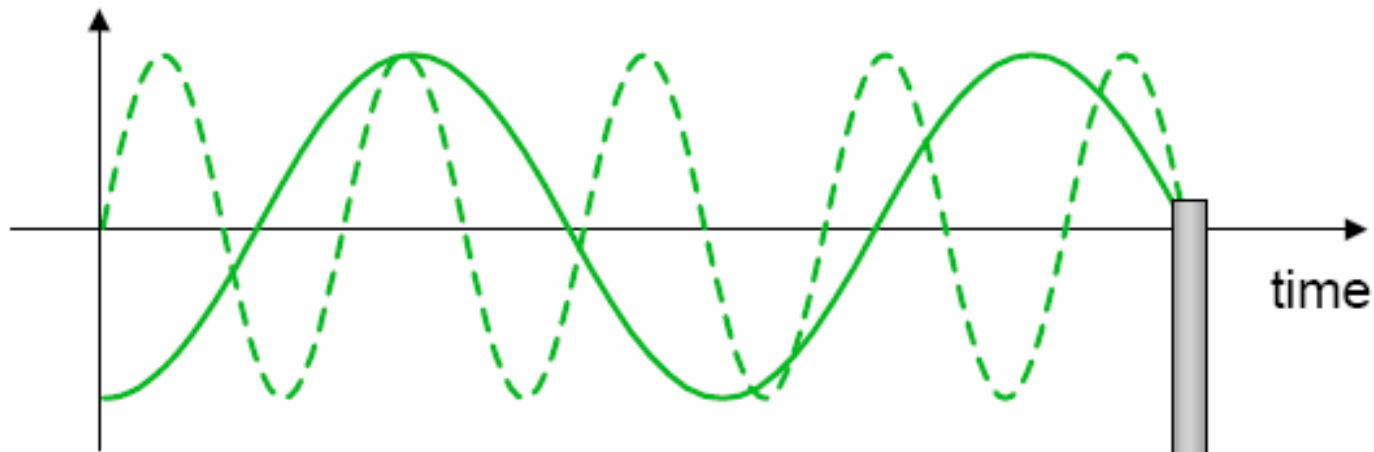


Period, T_n in [sec]

Frequency, $f_n = \frac{1}{T_n}$ in [Hz = 1/sec]

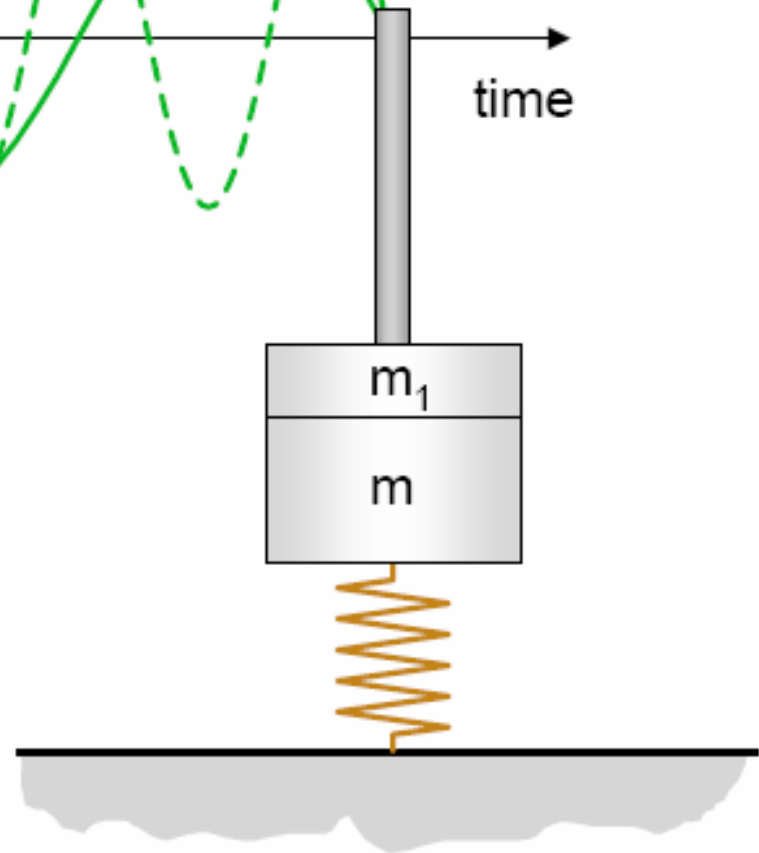
$$\omega_n = 2 \pi f_n = \sqrt{\frac{k}{m}}$$

Mass and Spring (Undamped Vibration)

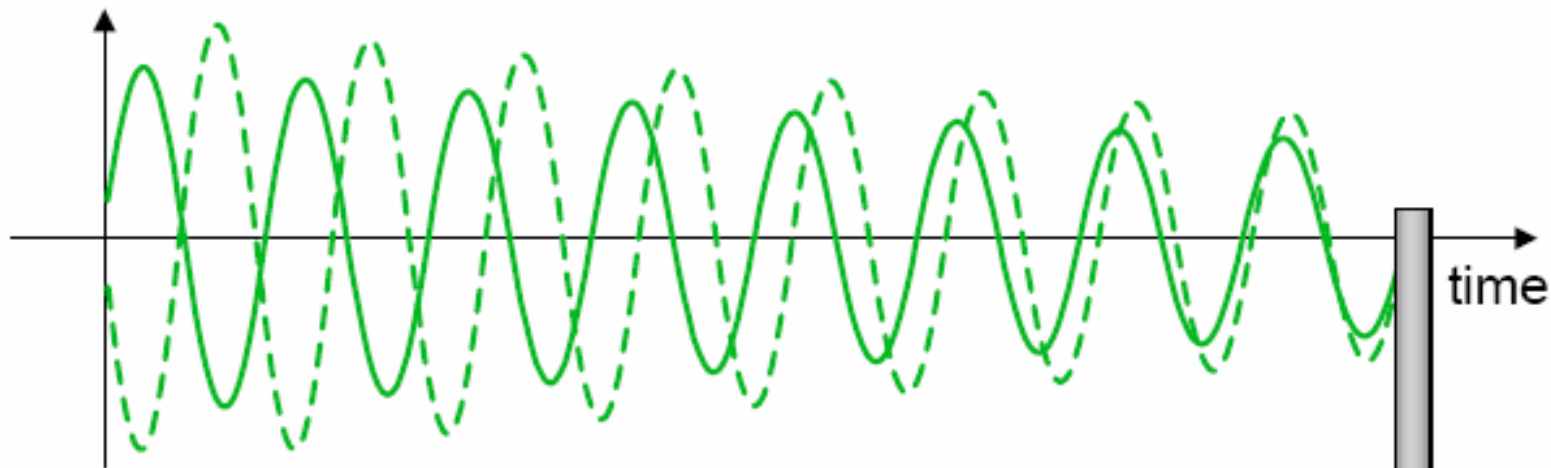


$$\omega_n = 2\pi f_n = \sqrt{\frac{k}{m + m_1}}$$

Increasing mass
reduces frequency

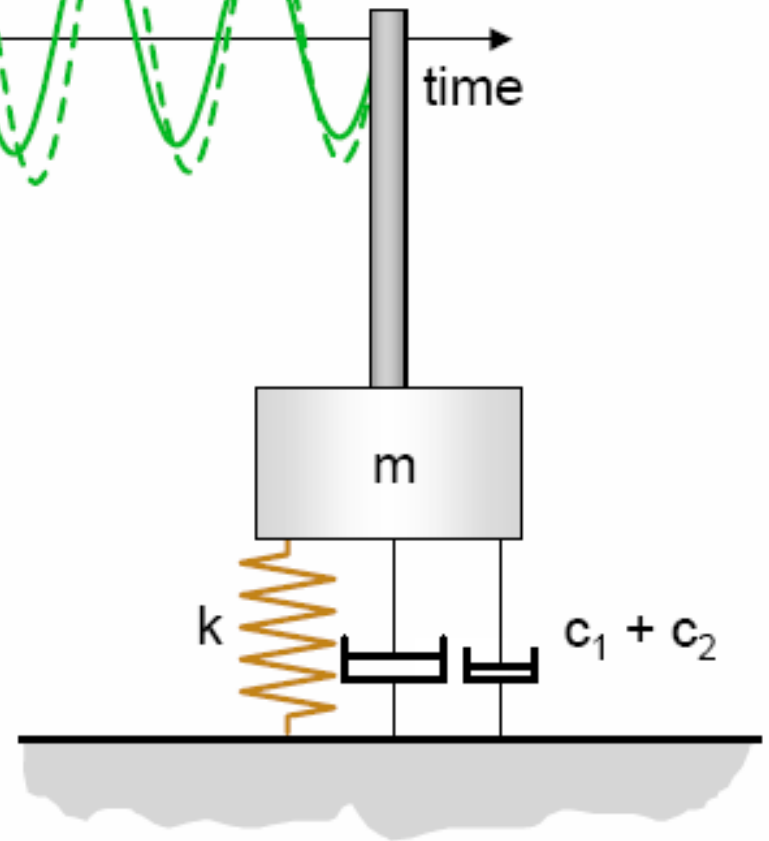


Mass, Spring and Damper (Damped Vibration)

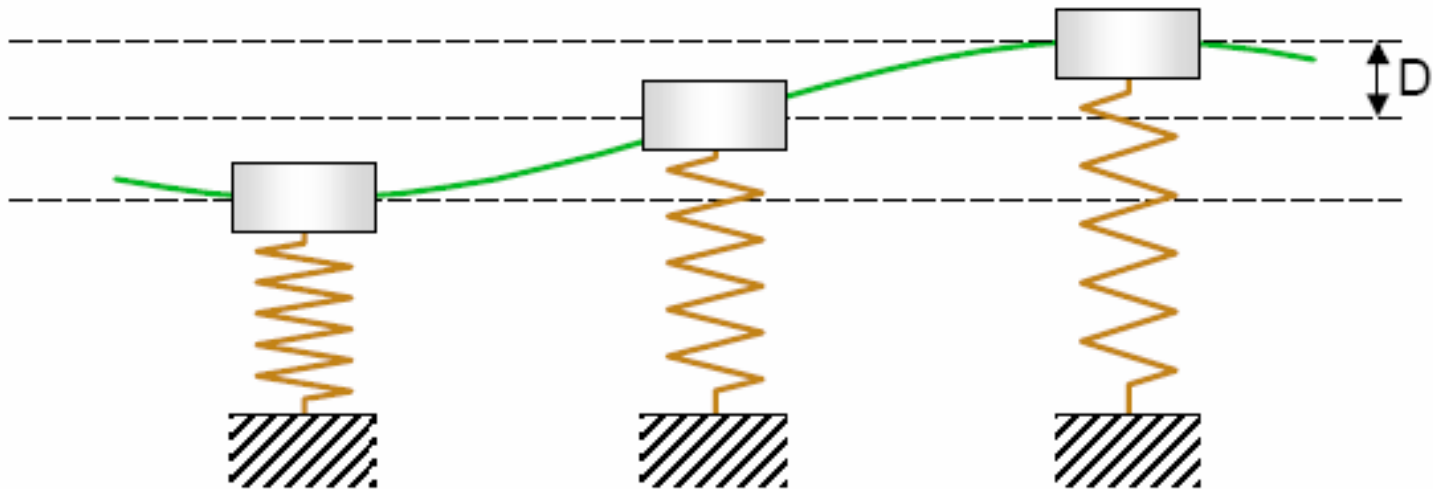


Increasing damping
reduces the amplitude

$$m\ddot{x} + c\dot{x} + kx = 0$$



Free Vibration



Energy transfer between Kinetic and Potential Energy
(assuming no damping)

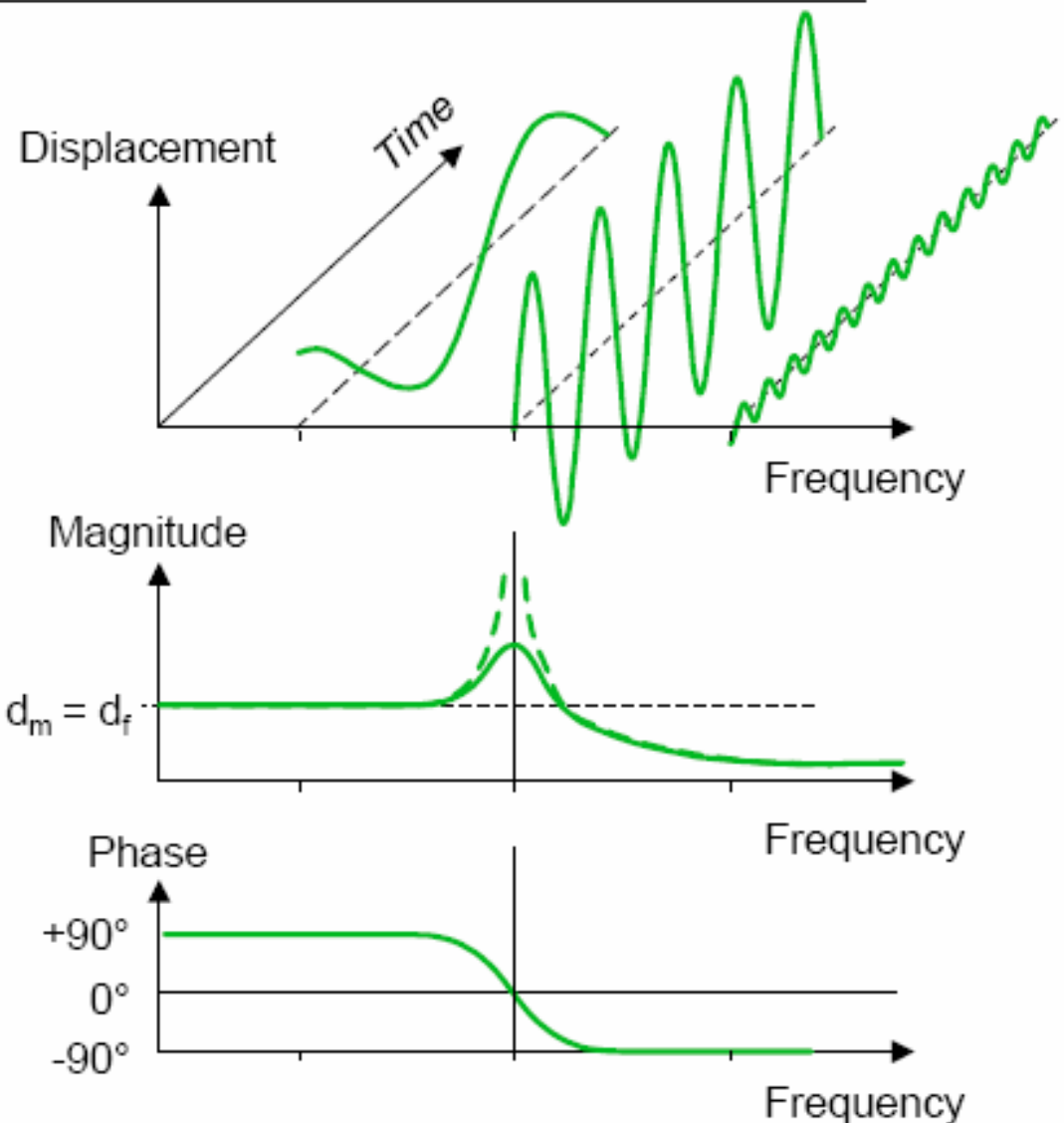
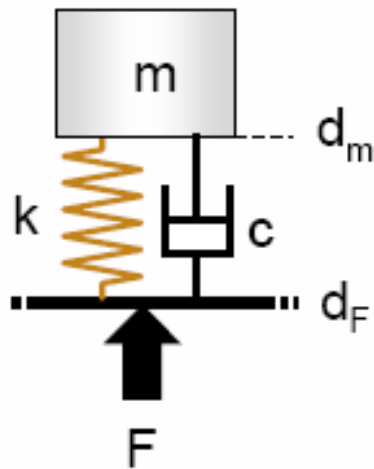
$$\Delta \text{ Kinetic Energy} = - \Delta \text{ Potential Energy}$$

$$\frac{1}{2} m V^2 = \frac{1}{2} k D^2, \text{ and } V = (2\pi f_n)D$$

$$\frac{1}{2} m (2\pi f_n)^2 D^2 = \frac{1}{2} k D^2$$

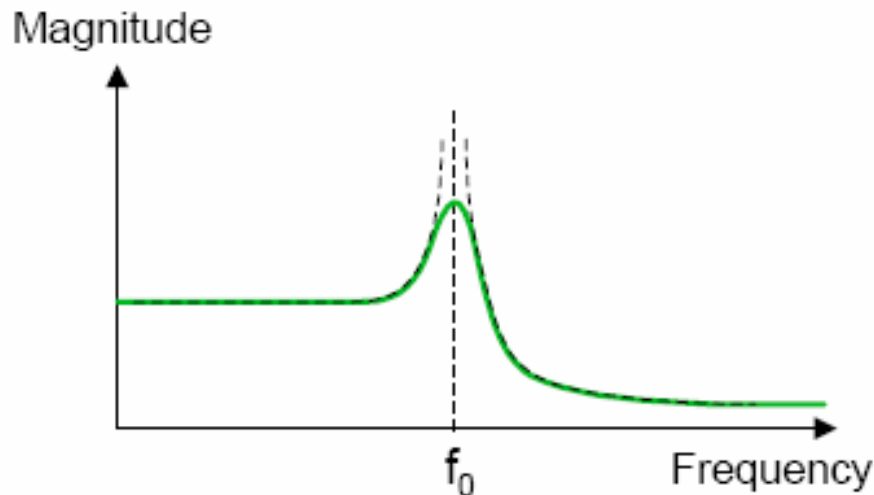
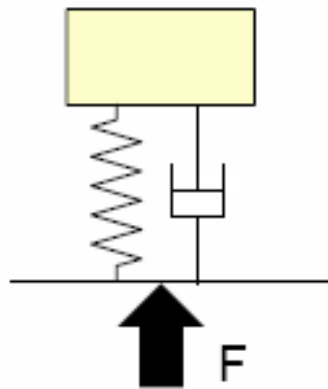
$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Forced Vibration

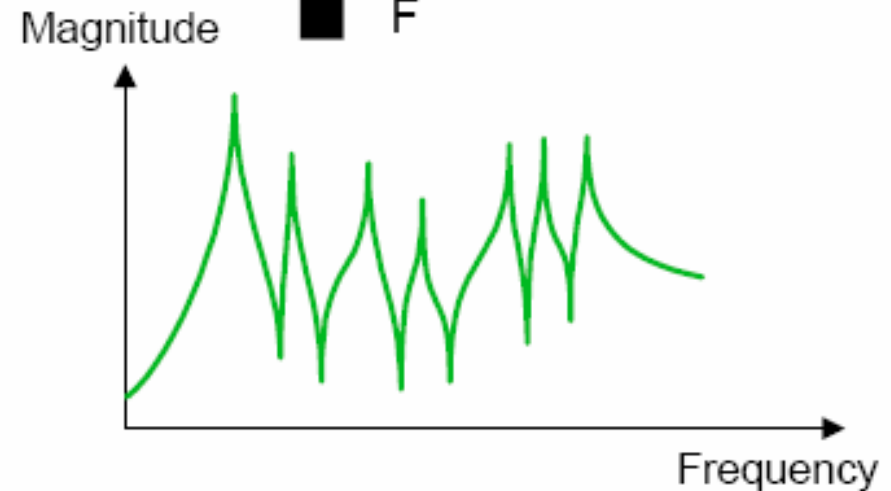
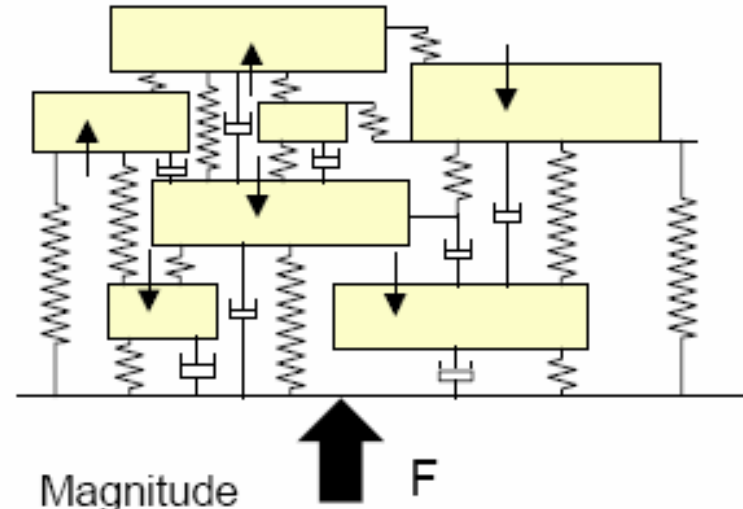


Response Models

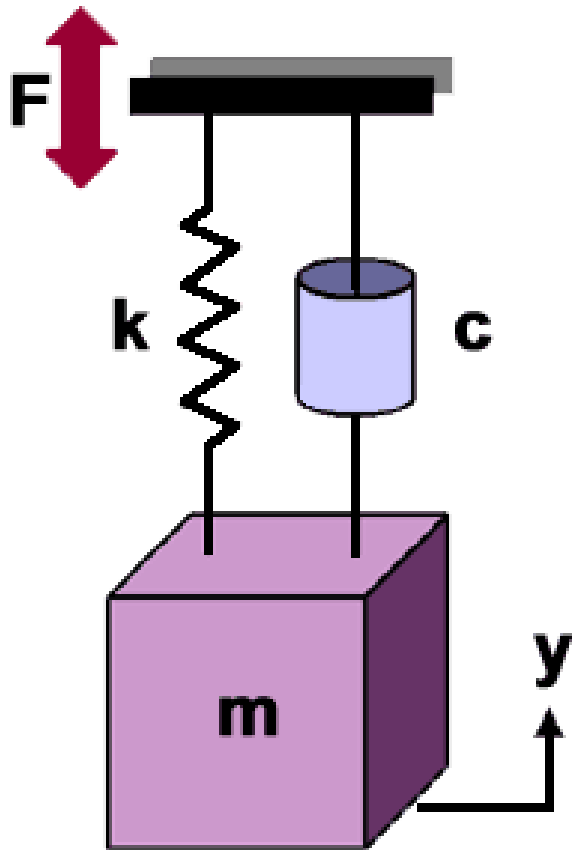
Single Degree of Freedom
SDOF



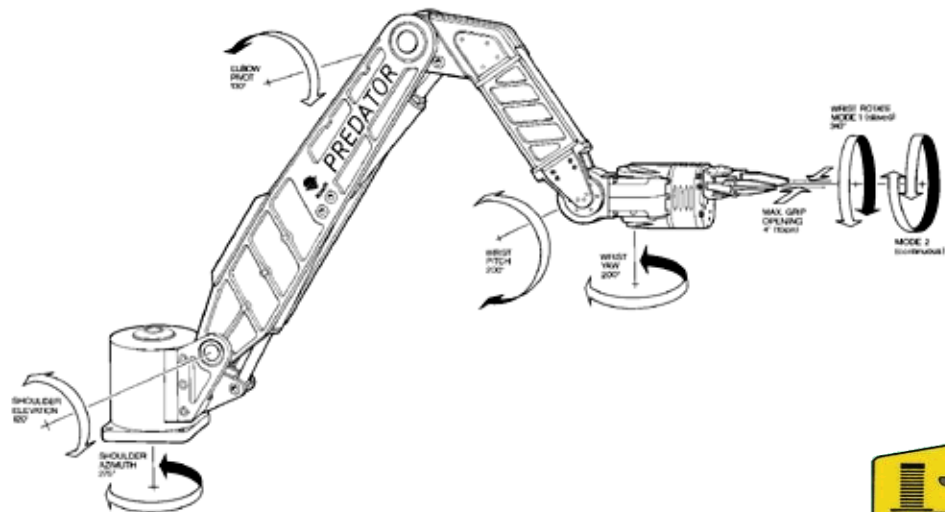
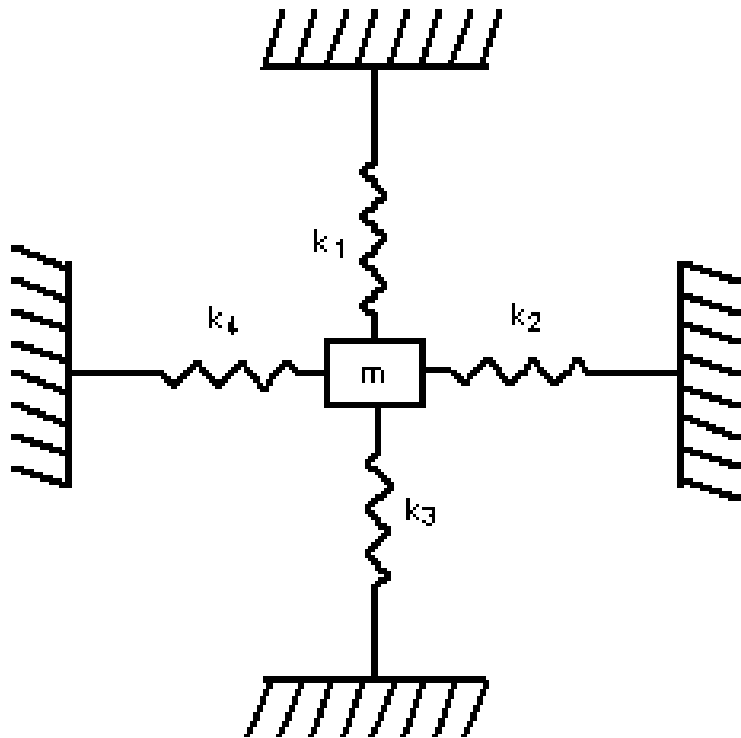
Multi Degree of Freedom
MDOF



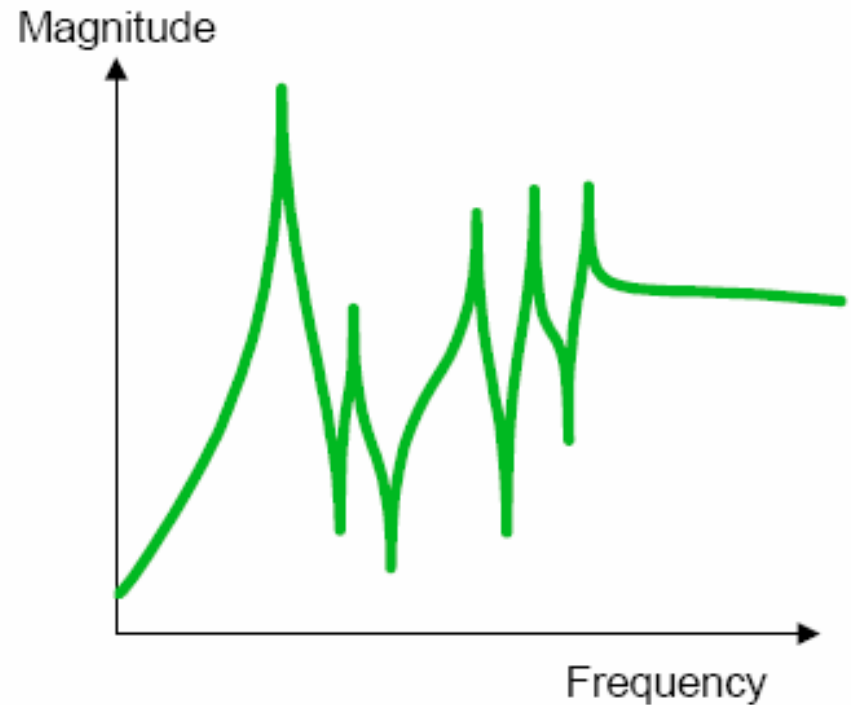
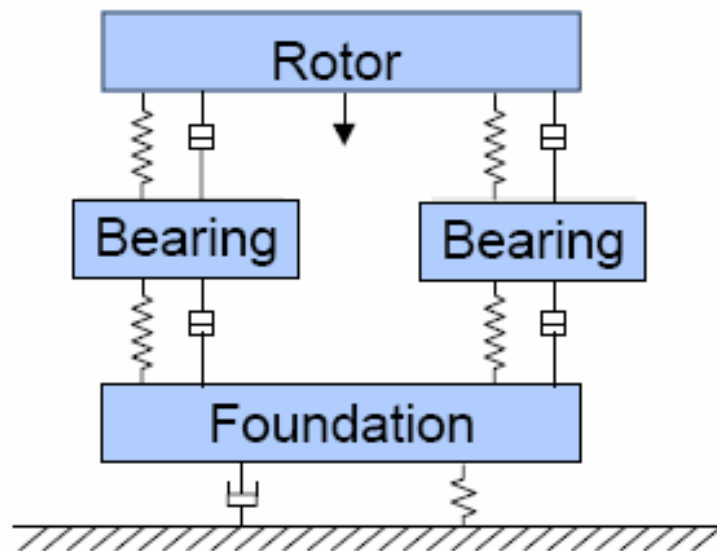
Single Degree of Freedom (SDOF)



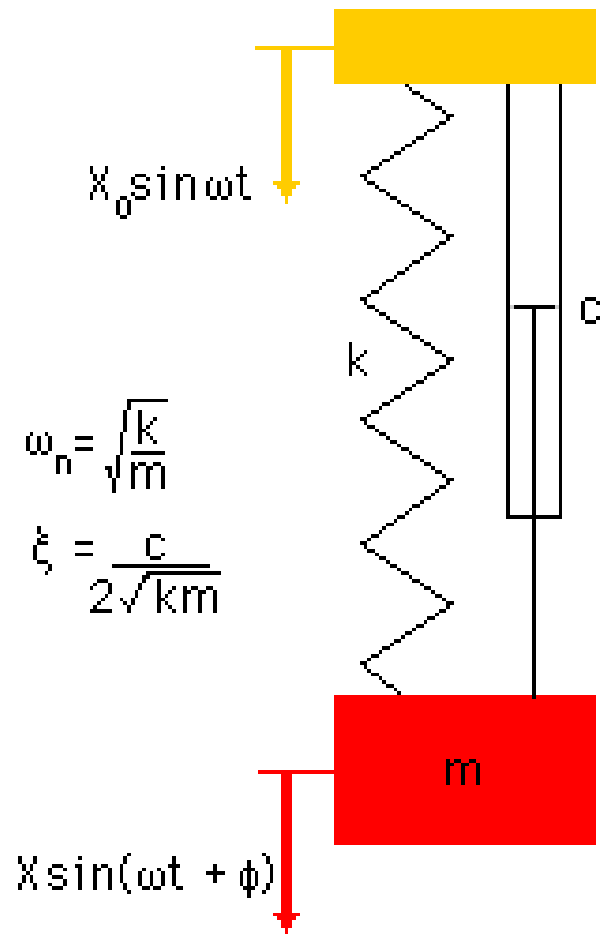
Multiple Degree of Freedom (MDOF)



“Real-world” Response

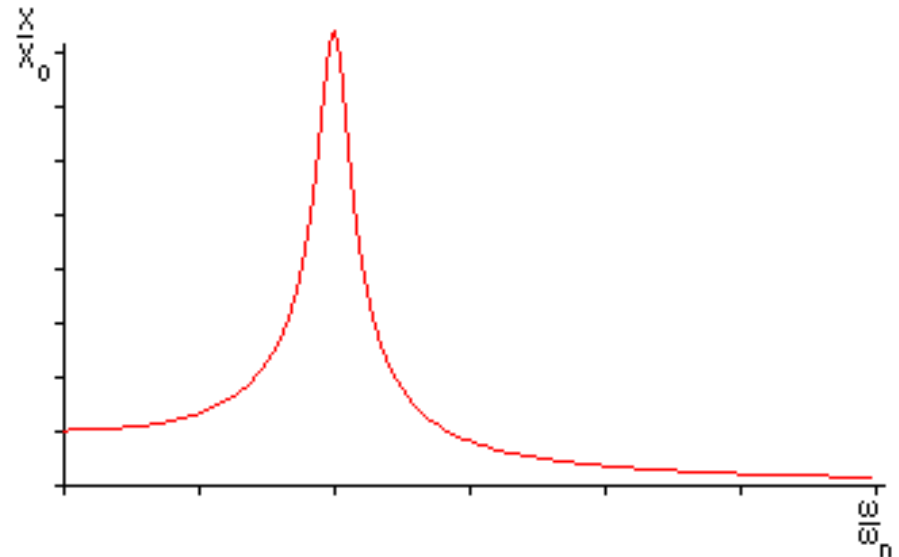


Transmissibility

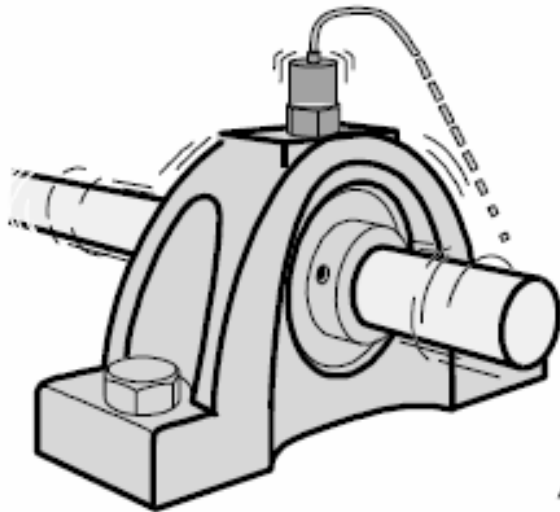


Steady State Excitation

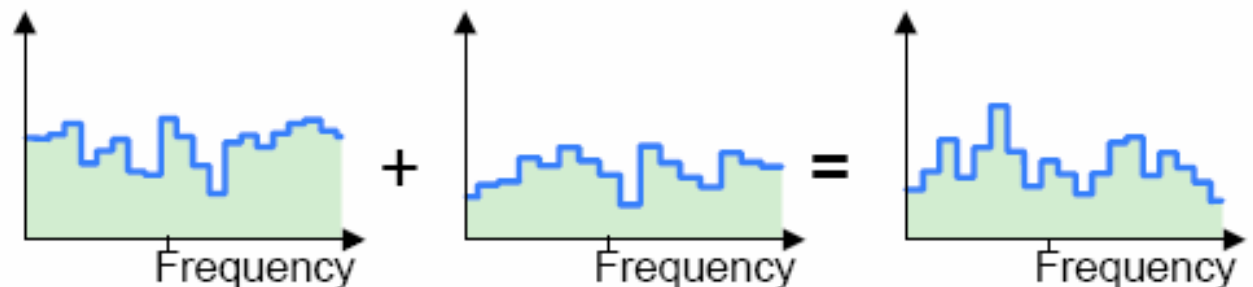
$$\frac{x}{x_0} = \frac{\left[1 + 4\xi^2 \left[\frac{\omega}{\omega_n} \right]^2 \right]^{1/2}}{\left[\left[1 - \left[\frac{\omega}{\omega_n} \right]^2 \right]^2 + 4\xi^2 \left[\frac{\omega}{\omega_n} \right]^2 \right]^{1/2}}$$



Forces and Vibration



Input Forces + System Response (Mobility) = Vibration



Forces caused by

- Imbalance
- Shock
- Friction
- Acoustic

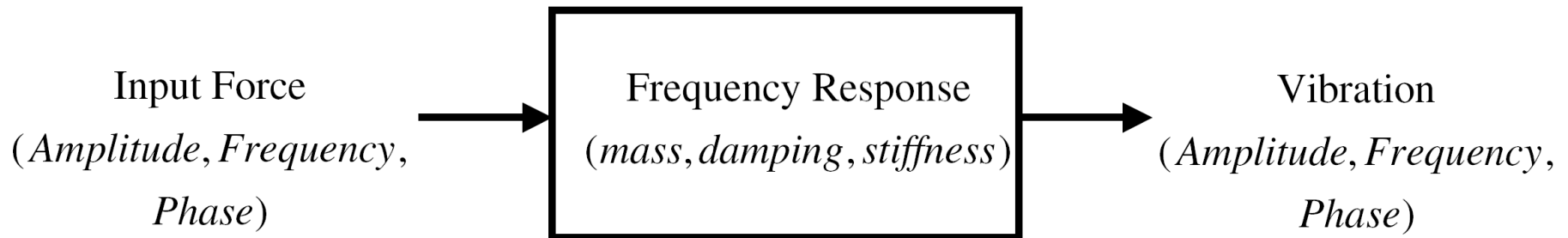
Structural Parameters:

- Mass
- Stiffness
- Damping

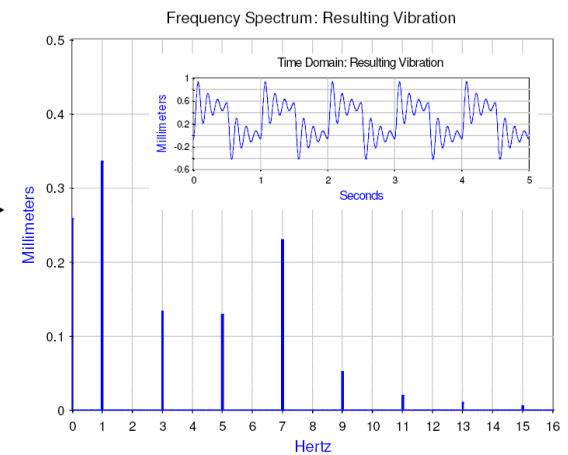
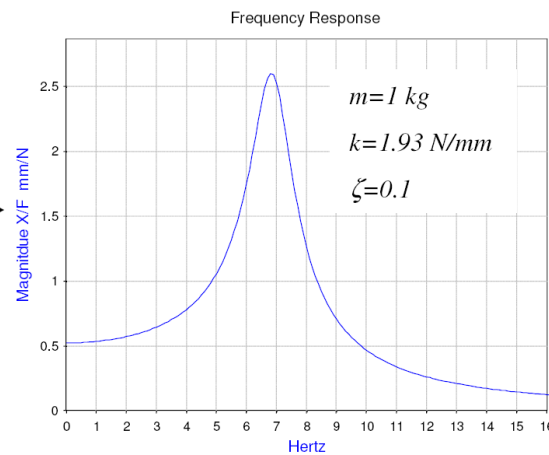
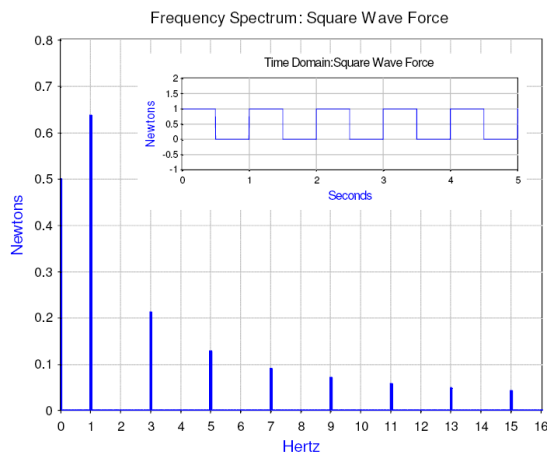
Vibration Parameters:

- Acceleration
- Velocity
- Displacement

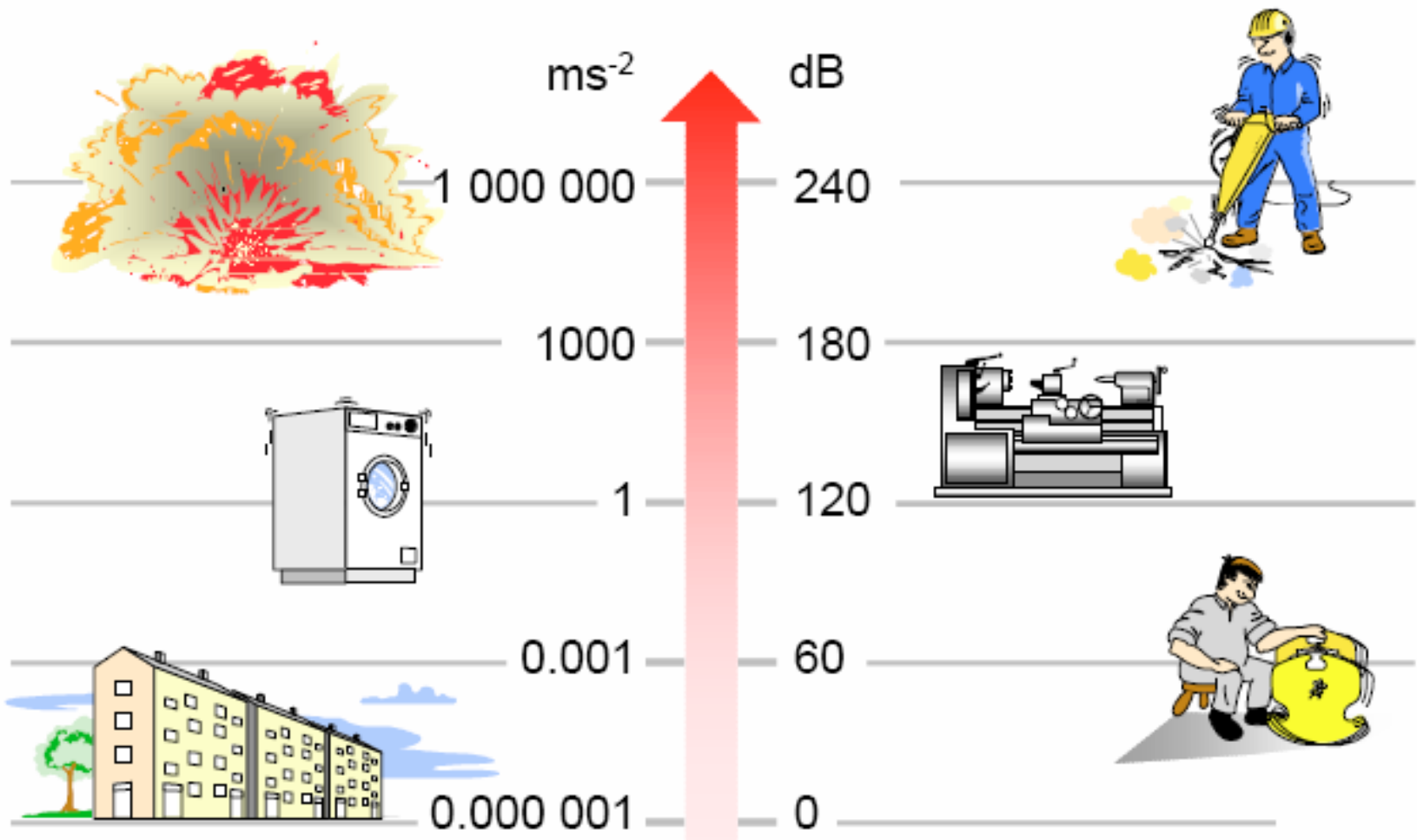
Frequency Response Function



$$F(\omega) \times H(\omega) = X(\omega)$$



“Real World” Vibration Levels



Why Do We Measure Vibration?



- To verify that frequencies and amplitudes do not exceed the material limits (e.g. as described by the Wöhler curves)
- To avoid excitation of resonances in certain parts of a machine
- To be able to dampen or isolate vibration sources
- To make conditional maintenance on machines
- To construct or verify computer models of structures (system analysis)

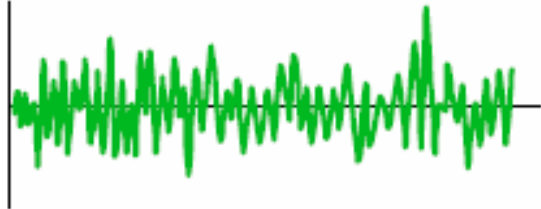
How do We Quantify Vibration?

- We make a measurement
- We analyse the results (levels and frequencies)

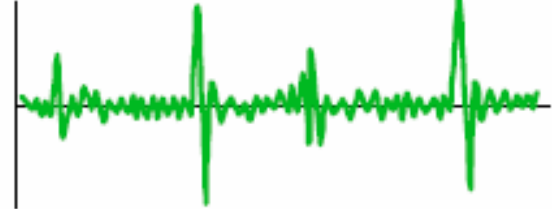
In order to make the analysis, we must first talk about the types of vibration signals we might encounter and how we measure these signals

Types of Signals

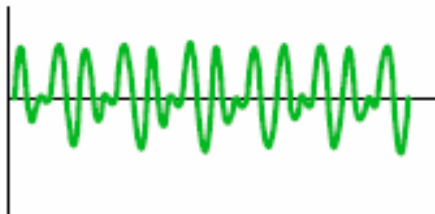
Stationary signals



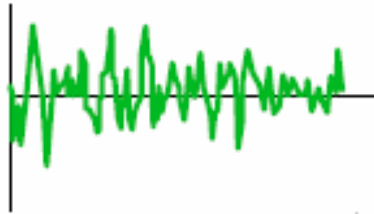
Non-stationary signals



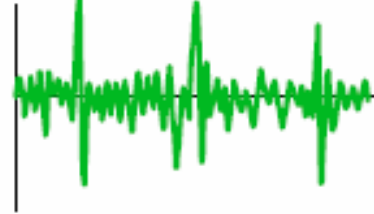
Deterministic



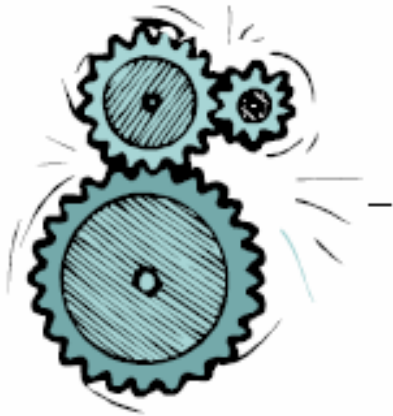
Random



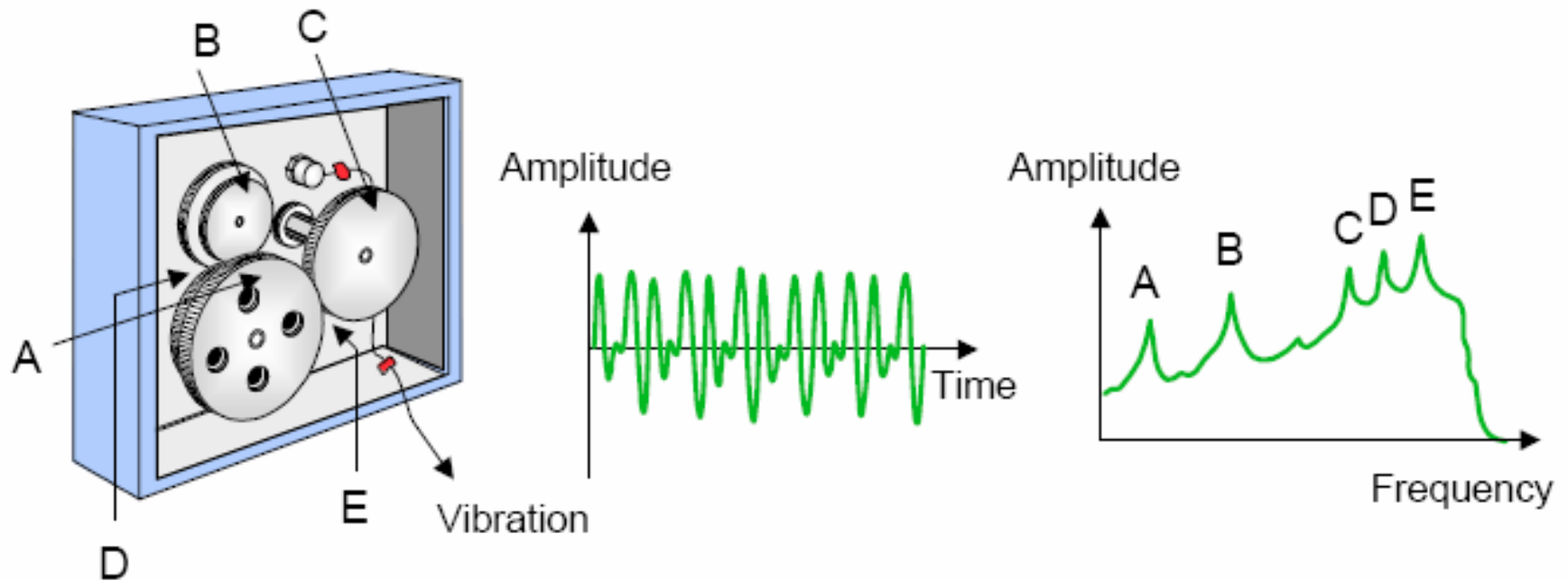
Continuous



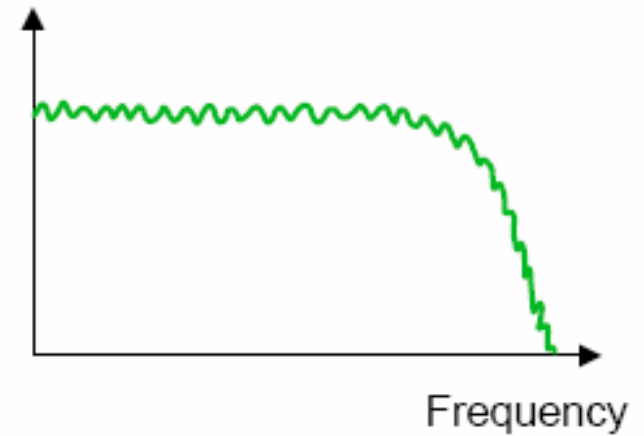
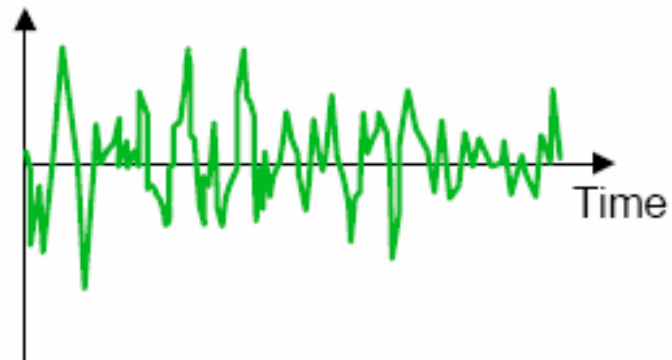
Transient



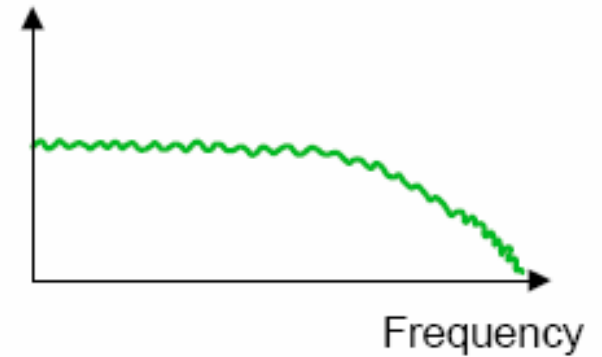
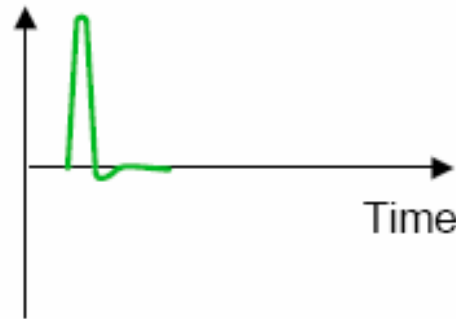
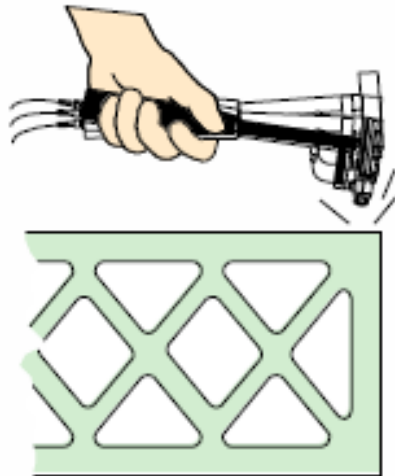
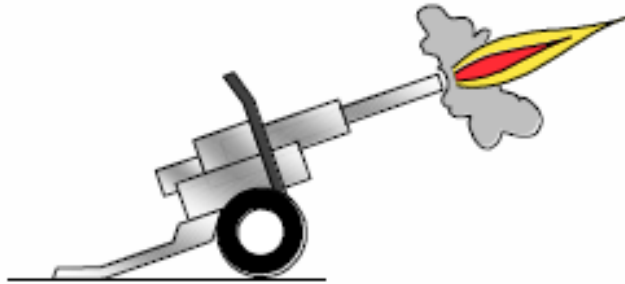
Deterministic Signals



Random Signals



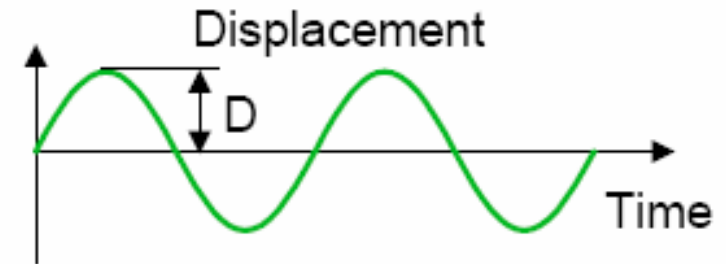
Impact-Impulse-Shock Signals



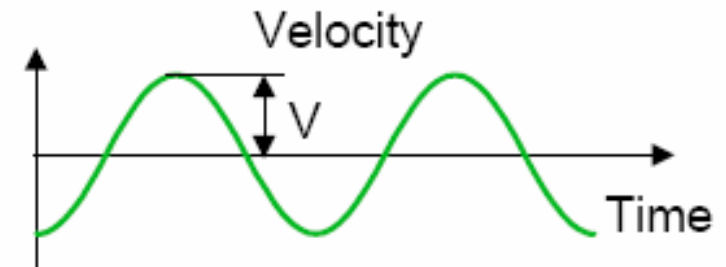
Linear vs. Oscillatory Motion



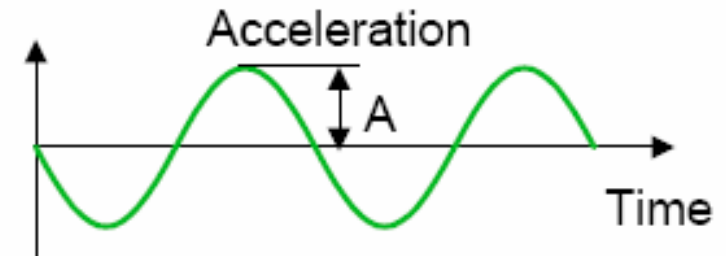
Detroit
35 Miles



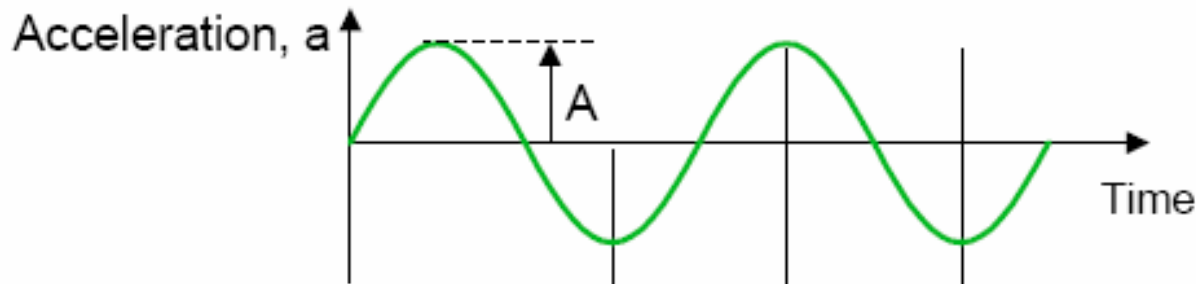
Speed
limit
65 MPH



TEST
0-60 MPH
in 8.6
second

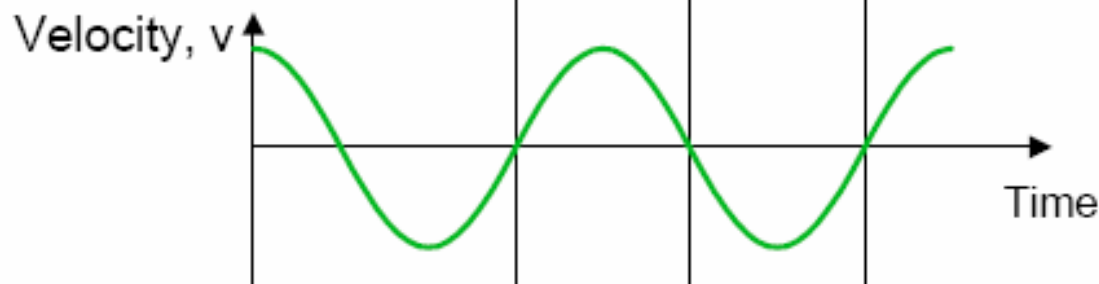


Conversion from Acceleration to Displacement



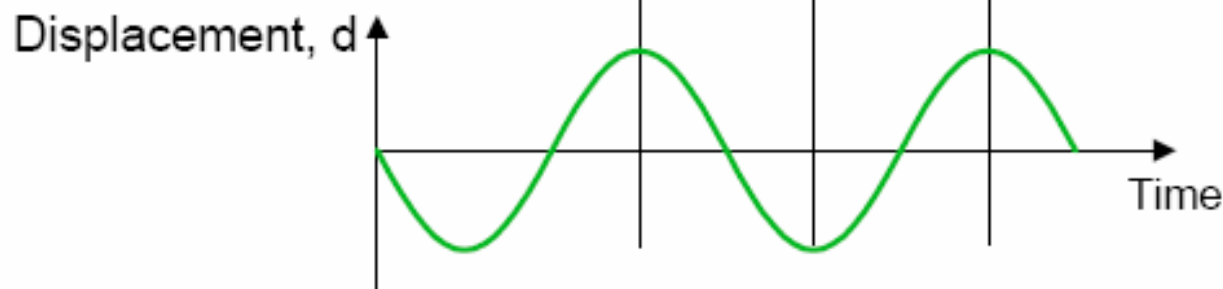
$$a = A \sin \omega t$$

$$a = A$$



$$v = \int a \, dt = -\frac{A}{\omega} \cos \omega t$$

$$v = \frac{A}{\omega} = \frac{A}{2\pi f}$$



$$d = \iint a \, dt \, dt = -\frac{A}{\omega^2} \sin \omega t$$

$$d = \frac{A}{\omega^2} = \frac{A}{4\pi^2 f^2}$$

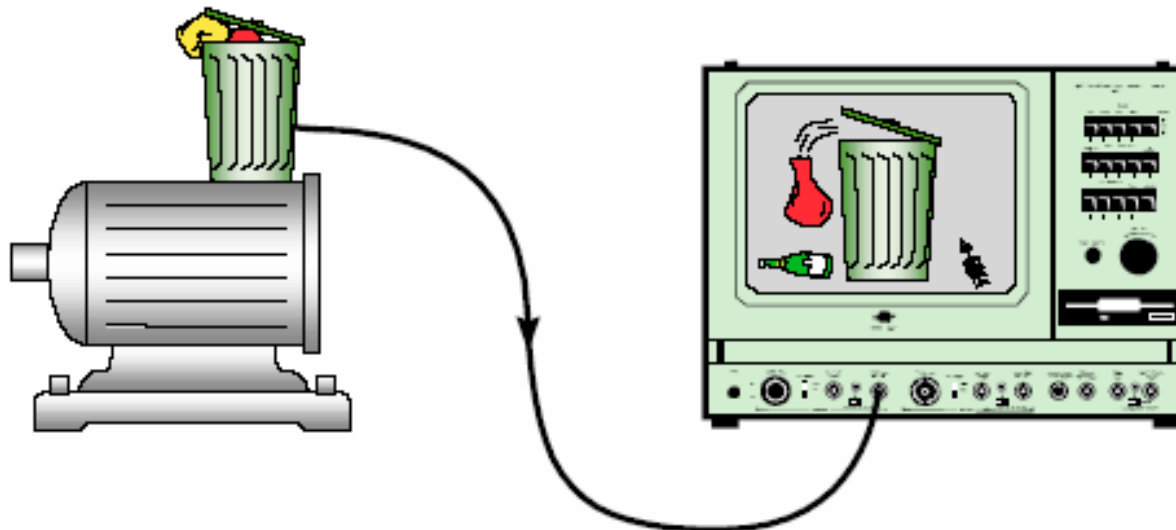
Units of Vibration Signals

Acceleration a	$1\text{ms}^{-2} \text{ (m/s}^2\text{)}$	$= 0.102g = 39.4 \text{ in/s}^2$
Velocity v	$1\text{ms}^{-1} \text{ (m/s)}$	$= 3.6 \text{ km/h} = 39.4 \text{ in/s}$
Displacement d	1m	$= 1000 \text{ mm} = 39.4 \text{ in}$

$$1g \equiv 9.80665 \text{ ms}^{-2}$$

GIGO

Garbage In = Garbage Out



Case History: Vibrations in a Gantry Crane, Unbalancing

• Problem

Very heavy vibrations were occurring in the crane's gantry structure during operation. The production management were in a great dilemma, a production stoppage for investigation and remedial action would be very costly, while a breakdown would be catastrophic.

• Source identification

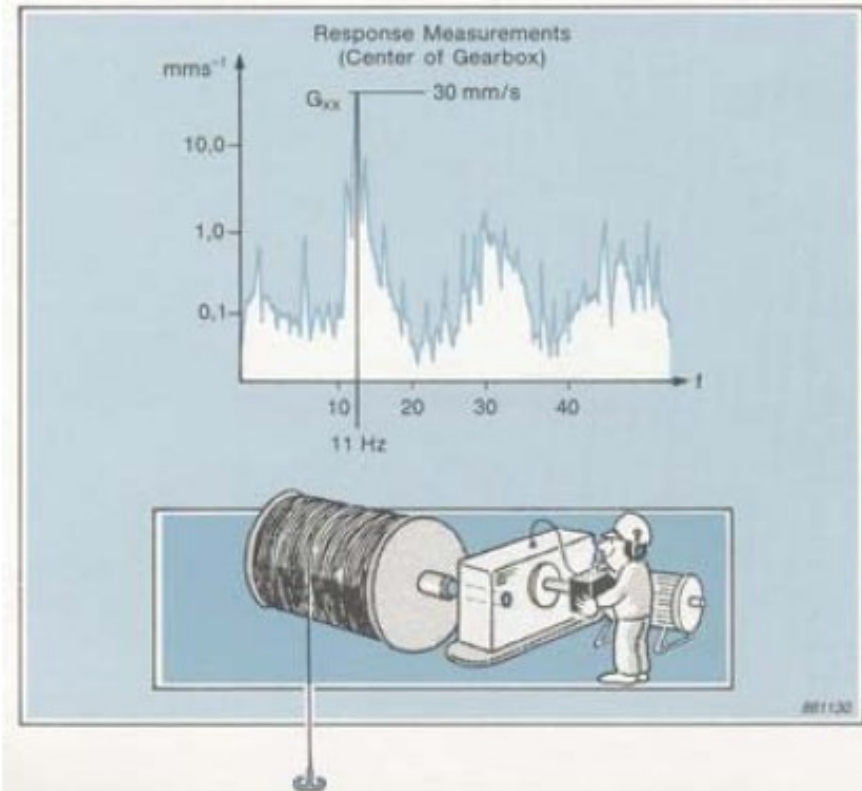
The vibrations were only present when a particular winch unit was involved in a hoisting operation. From a few vibration measurements, the source was easily identified as the gearbox in that unit. Spectrum analysis of measurements on the gearbox showed that the predominant vibration frequency was 11 Hz. This frequency was, in turn, traced to the intermediate gearwheel, corresponding to its rotational frequency.

• Problem identification

The problem now was: were the force levels generated by the gearbox too high? Or was it a normal force level amplified by a resonance in the structure?

To determine the answer, a driving point mobility measurement was made at the shaft bearing of the gearwheel in question. Excitation by a large impactor on the top of the gearbox made the measurement both fast and easy.

The FRF showed no resonance at the observed vibration frequency of 11 Hz, and the source was diagnosed to be forced vibrations due to rotating unbalance.



• Determination of the unbalance forces

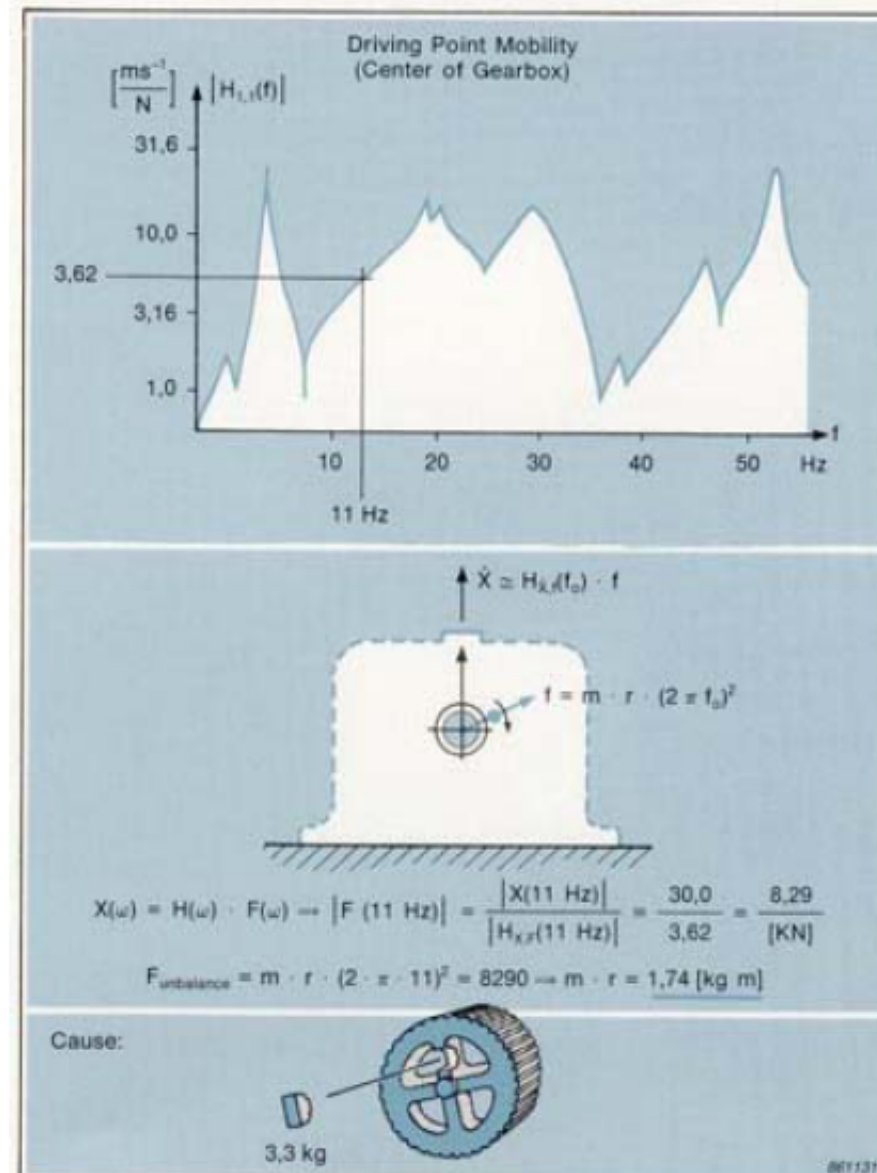
A straightforward technique was applied to determine the mass unbalance forces. Treating the shaft bearing as a single input - single output system, we can rewrite our linear model:

$$F(\omega) = \frac{X(\omega)}{H(\omega)}$$

This was solved for the magnitudes, at the frequency of 11 Hz. The unbalance force magnitude was found to be 8,29 kN. A further calculation showed that this was equal to a mass moment of 1,74 kg m.

• Solution

A balancing shop was alerted, and production work was planned to proceed without crane operations during one working shift. The gearbox was dismantled and the gear-wheel transported to and from the balancing shop. Everything was remounted and ready for trouble-free operation within eight hours. An interesting point is that, although the assumption of a single input - single output model is coarse, the predicted mass moment of the unbalance was almost exact. It had been caused by a fracture discharging a piece of the casting, the weight of the fragment was 3,3 kg, and its centre of gravity was 0,53 m from the centre of the shaft.



Vibration Isolation Pads

- Vibration Isolation Pads are high grade neoprene isolation mediums which can efficiently and economically control structure-born noise and resonant vibration.
- The most common applications have them placed under machinery, grinders, compressors, metal panel enclosures or other common sources of high intensity noise levels for vibration and shock control.

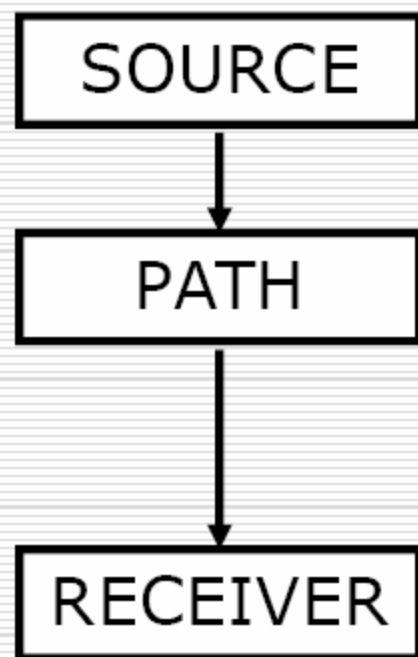
Vibration Isolation

- The Source – Path – Receiver Model
- Sources of vibration
 - Fans
 - Pumps
 - Compressors & Chillers
 - Cooling Towers
- Receiver's Tolerance for Vibration
- The Engineering Problem, and why we need Vibration Isolation

Vibration Isolation

- ❑ Vibration Transmission
- ❑ Selecting the Static Deflection of Isolators
- ❑ Your Master Specification (Puh-LEEZ!)
- ❑ Isolator Types
 - Pads
 - Mounts
 - Springs
 - Combinations
- ❑ Applications

Components of Vibration Transmission



Machinery Generating Vibration

- ☞ Cooling Towers, Motors, Fans, Pumps, Compressors
- ☞ Turbulent flow in pipe and duct

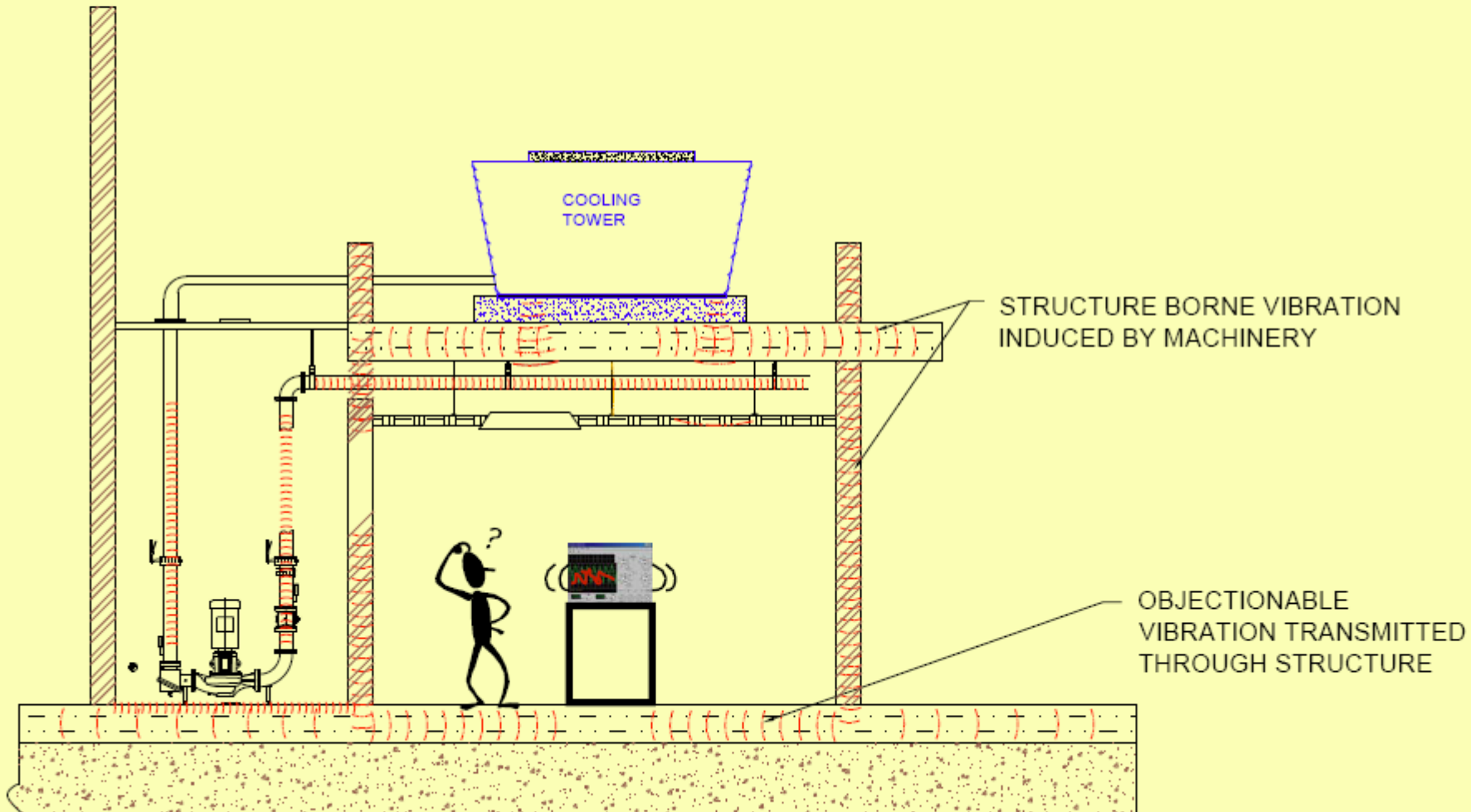
Medium Through Which Vibration is Transmitted

- ☞ Most structural building components (floors, beams, columns, walls, etc)
- ☞ Pipe
- ☞ An isolation system is used to "block" the path

The Occupant

- ☞ Building owners and tenants **demand** comfortable work environments. Quality of workplace affects worker production
 - ☞ Quality of classroom affects student learning
 - ☞ High - tech equipment in hospitals or laboratories
-

Introduction to Source-Path-Receiver Model



Vibration Sources

- ☐ Fans and Air Handlers
 - ☐ Pumps
 - ☐ Compressorized Equipment
 - ☐ Cooling Towers
-

Fans and Air Handlers

- ❑ Balance Standard per AMCA Fan Applications Manual, Standard 204-05
- ❑ Most fan and vent products are *not factory tested*.
- ❑ The factory vibration tests are reserved for large centrifugal, vaneaxial, and mixed flow fans.



Fans and Air Handlers – Factory Test

Vibration Velocity, Measured at Fan RPM

Fan Application Category	Rigidly Mounted Vibration Velocity (in/sec, peak)	Flexible Mounted Vibration Velocity (in/sec, peak)
BV-1	0.50	0.60
BV-2	0.20	0.30
BV-3	0.15	0.20
BV-4	0.10	0.15
BV-5	0.08	0.10

BV-4 and
BV-5 require
field trim

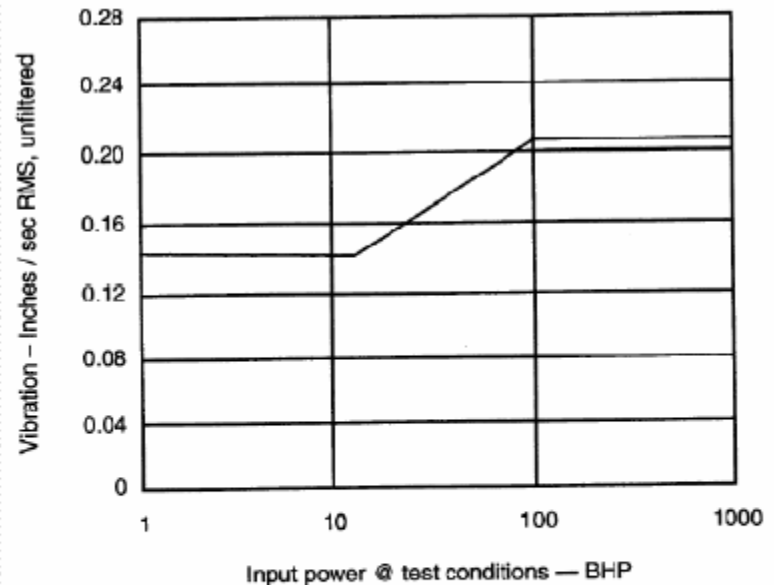
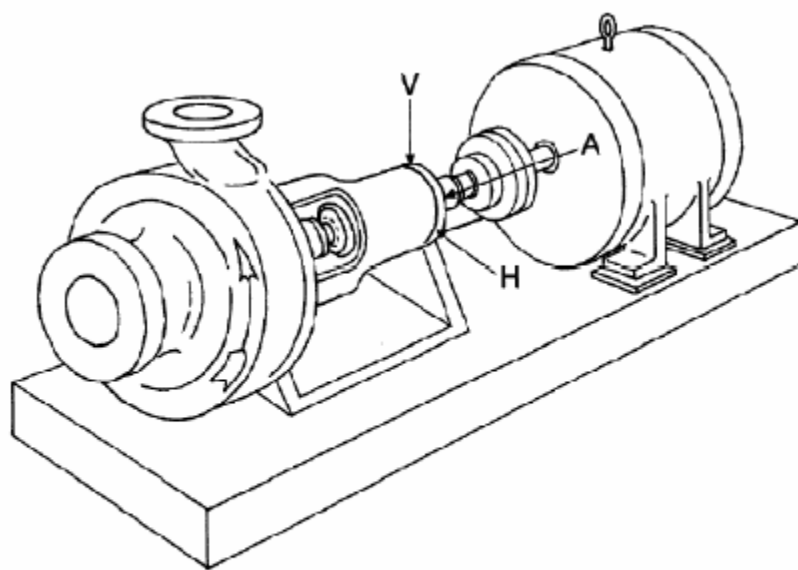


Direct-Drive Fans Can Achieve Lower Vibration Levels than Belt-Drive Fans in a factory test.

Field measured vibration ranges 25-50% higher than factory tested

Pump Vibration

- Reference: ANSI / Hydraulic Institute Standard 9.6.4
- Expect a range of 0.14 to 0.21 in/sec RMS vibration velocity, depending on horsepower



HVAC Compressors

- ❑ ASHRAE 47.39, Table 47, “Maximum Allowable RMS Velocity Levels”
- ❑ Centrifugal compressors 0.13 in/sec

Table 41 Maximum Allowable RMS Velocity Levels

Equipment	Allowable rms Velocity, mm/s
Pumps	3.3
Centrifugal compressors	3.3
Fans (vent sets, centrifugal, axial)	2.3

Cooling Towers

- ❑ Industry Standard – Goal 0.3 inches per second vibration velocity
 - ❑ Set vibration switches at 1.5 x design velocity \sim 0.45 inches per second vibration indicates a problem that requires resolution.
 - ❑ Operating range 0.25 – 0.35 inches per second
-

Summary of Vibration Sources

Vibration Source	Vibration Velocity, in/sec	Comments
Fans	0.20 – 0.30 Peak	BV-2 to BV-3, factory tested
Pumps	0.14 – 0.21 RMS	Varies with horsepower and style
Compressors	0.13	ASHRAE reference
Cooling Towers	~0.25 – 0.35 RMS	General rule of thumb
Full Range (All products above)	0.13 – 0.35	<i>This is the vibration level that HVAC equipment is supposed to emit!</i>

Severe structural damage occurs at 2 inches/second velocity (1.14 mph)

Plaster and facades damaged at 1 inch/second velocity (0.57 mph)

Acceptable Occupancy Limits

Environment	Acceptable Vibration, in/sec	Vibration Reduction from 0.3 in/sec	Vibration Reduction from 0.15 in/sec
Office	0.016	94.7%	89.3%
Residence (night) or Class A Office	0.006	98.0%	96.0%
Operating Room	0.004	98.7%	97.3%
Lab w/Microscope, 400x Magnification	0.002	99.3%	98.7%
Eye surgery / neurosurgery	0.001	99.7%	99.3%
MRI to 1mm detail	0.0005	99.8%	99.7%
Electron Microscope, 30,000x Magnification	0.00025	99.9%	99.8%

Vibration Criteria

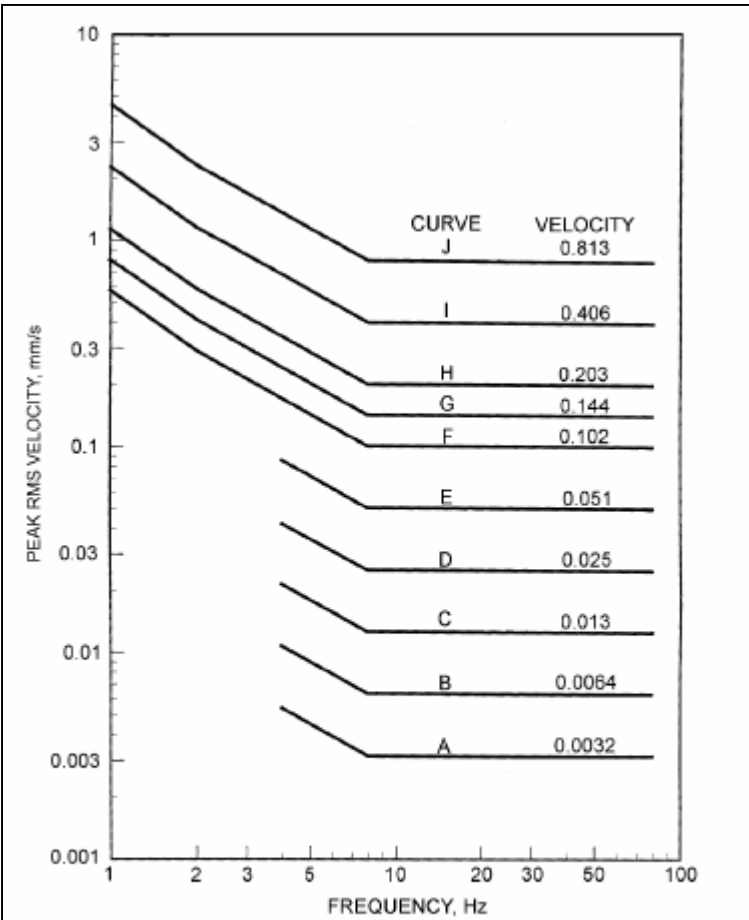


Fig. 39 Building Vibration Criteria for Vibration Measured on Building Structure

Table 40 Equipment Vibration Criteria

Human Comfort	Time of Day	Curve ^a
Workshops	All	J
Office areas	All ^b	I
Residential (good environmental standards)	0700-2200 ^b	H
	2200-0700 ^b	G
Hospital operating rooms and critical work areas	All	F

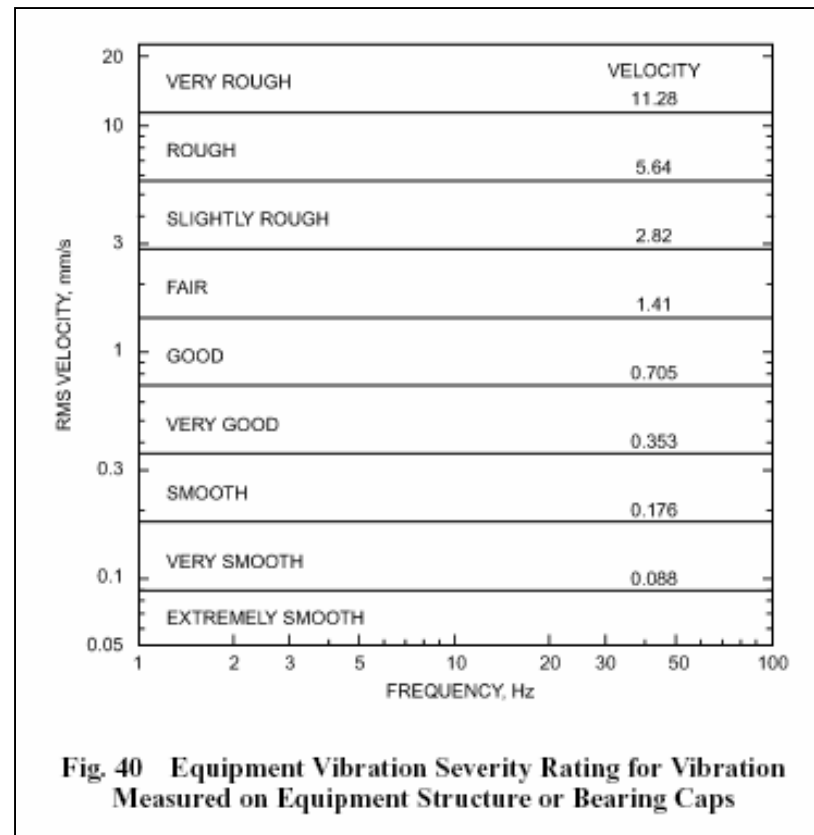


Fig. 40 Equipment Vibration Severity Rating for Vibration Measured on Equipment Structure or Bearing Caps

Rats! Now what?

- ❑ The acceptable vibration level is 0.1% to 10% of the source level.
- ❑ Something must be placed in the vibration path between source and receiver
- ❑ Structural elements are at best transmitters (transmit vibration perfectly), and at worst resonators (amplify vibration)
- ❑ That "something" is called a "vibration isolator" and is generally resilient instead of rigid

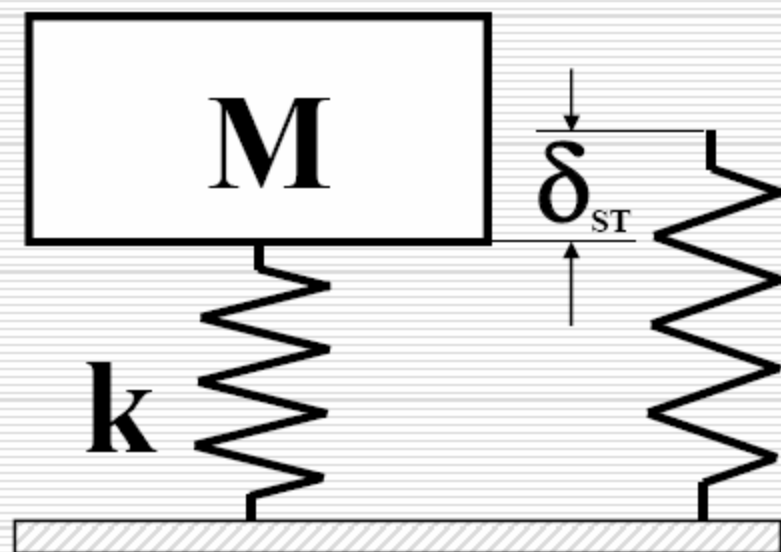


Percentage of Vibration Transmission

Required Static Deflection of Isolator, inches
--

<i>Operating</i> Speed, rpm	0.5%	1%	5%	10%	25%
3600	0.55	0.27	0.06	0.03	0.01
1800	2.20	1.10	0.23	0.12	0.05
1200	4.90	2.50	0.52	0.27	0.12
900	8.80	4.40	0.92	0.48	0.22
600	n/a	9.90	2.10	1.10	0.49

Isolation Basics – From your Junior Year dynamics class when you were hung over



Static Forces:

$$F = k\delta = Mg$$

Small deflection:

- Stiff system
- High natural frequency

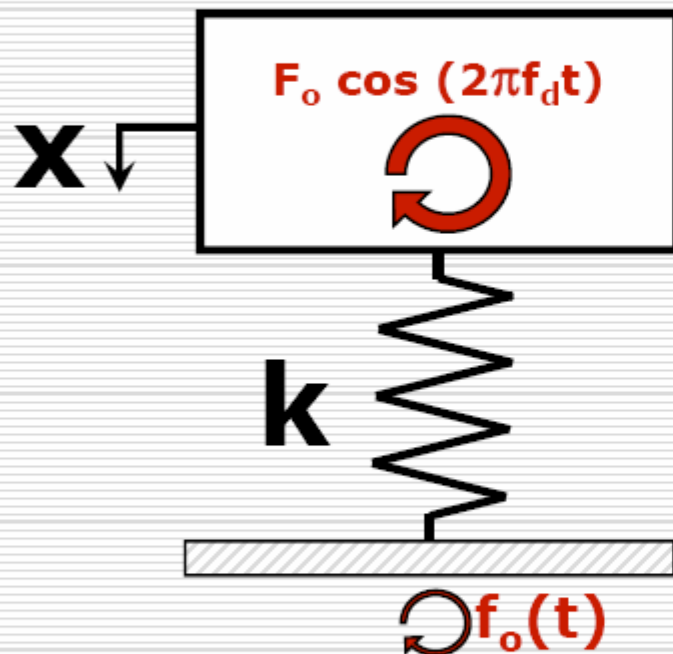
Large deflection:

- Compliant system
- Low natural frequency

This system has a ***natural frequency*** that it will vibrate at.

$$2\pi f_n = \sqrt{k / M} \qquad f_n = 3.13 \sqrt{(1/\delta)}$$

More Theory



Dynamic Forces:

$$F_o \cos(2\pi f_d t) = Mx'' + kx$$

**This unrestrained,
periodic force**

**Is resisted by inertia of
the object and the spring
at its current position**

- The goal is to minimize the ratio f_o / F_o
 - Called "Transmissibility"
 - $= 1 / (1 - f_d / f_n)^2$
-

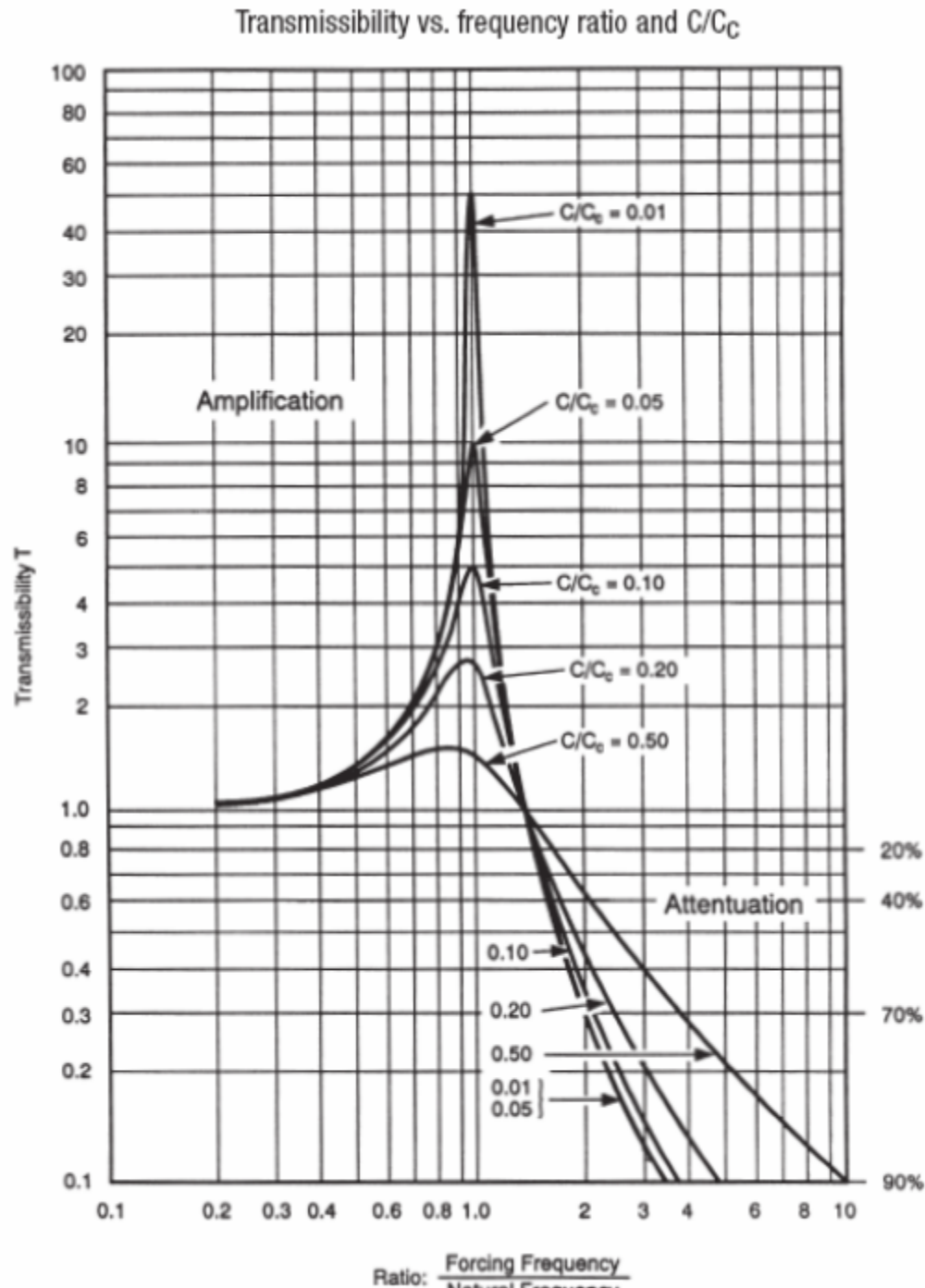
More Theory

Forcing Frequency is well below the Natural Frequency: no isolation or amplification.

Forcing Frequency is well above the Natural Frequency (minimum 3x): vibration isolation

Forcing Frequency is roughly equal to Natural Frequency:

Amplification / Resonance



Percentage of Vibration Transmission

Required Static Deflection of Isolator, inches					
--	--	--	--	--	--

<i>Operating</i> Speed, rpm	0.5%	1%	5%	10%	25%
3600	0.55	0.27	0.06	0.03	0.01
1800	2.20	1.10	0.23	0.12	0.05
1200	4.90	2.50	0.52	0.27	0.12
900	8.80	4.40	0.92	0.48	0.22
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Your Master Specification

- ❑ ASHRAE 2007 Applications
 - ❑ Chapter 47
 - ❑ Sheets 47.40 through 47.42
 - ❑ Gives general guidelines
 - ❑ Do not blindly refer to this document.
Contractors need a schedule of isolated equipment for clarity.
 - ❑ Separate specification for seismic jobs
-

Selection Guide for Vibration Isolation

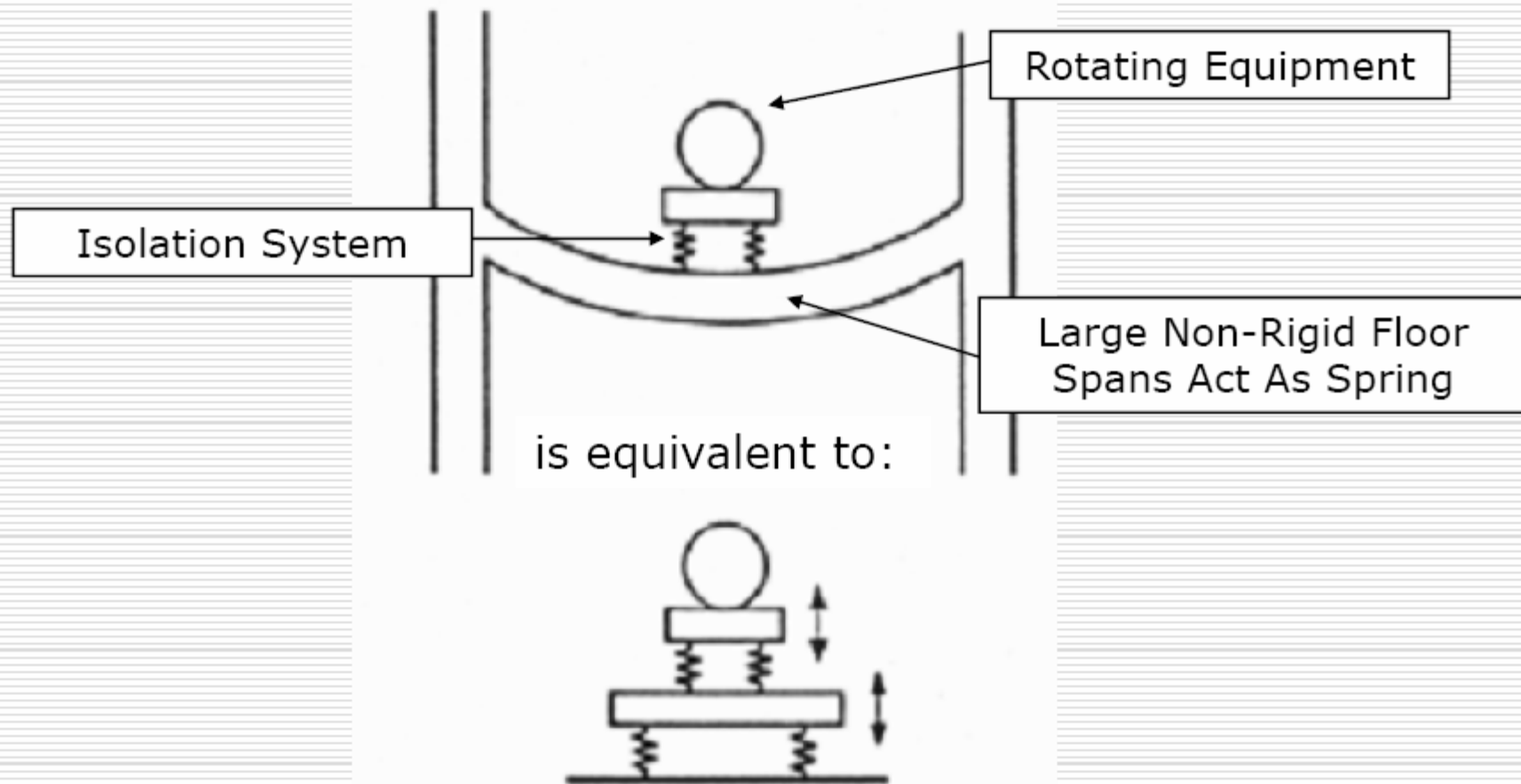
47.40

2007 ASHRAE Handbook—HVAC Applications

Table 48 Selection Guide for Vibration Isolation

			Equipment Location (Note 1)												Reference Notes
			Floor Span												
			Slab on Grade			Up to 20 ft			20 to 30 ft			30 to 40 ft			
			Base Isolator Type	Type	Min.	Base Isolator Type	Type	Min.	Base Isolator Type	Type	Min.	Base Isolator Type	Type	Min.	
					Defl., in.			Defl., in.			Defl., in.			Defl., in.	
Equipment Type	Horsepower and Other	RPM													
Refrigeration Machines and Chillers															
Reciprocating	All	All	A	2	0.25	A	4	0.75	A	4	1.50	A	4	2.50	2,3,12
Centrifugal, screw	All	All	A	1	0.25	A	4	0.75	A	4	1.50	A	4	1.50	2,3,4,12
Open centrifugal	All	All	C	1	0.25	C	4	0.75	C	4	1.50	C	4	1.50	2,3,12
Absorption	All	All	A	1	0.25	A	4	0.75	A	4	1.50	A	4	1.50	
Air Compressors and Vacuum Pumps															
Tank-mounted horiz.	≤10	All	A	3	0.75	A	3	0.75	A	3	1.50	A	3	1.50	3,15
	≥15	All	C	3	0.75	C	3	0.75	C	3	1.50	C	3	1.50	3,15
Tank-mounted vert.	All	All	C	3	0.75	C	3	0.75	C	3	1.50	C	3	1.50	3,15
Base-mounted	All	All	C	3	0.75	C	3	0.75	C	3	1.50	C	3	1.50	3,14,15
Large reciprocating	All	All	C	3	0.75	C	3	0.75	C	3	1.50	C	3	1.50	3,14,15
Pumps															
Close-coupled	≤7.5	All	B	2	0.25	C	3	0.75	C	3	0.75	C	3	0.75	16
	≥10	All	C	3	0.75	C	3	0.75	C	3	1.50	C	3	1.50	16
Large inline	5 to 25	All	A	3	0.75	A	3	1.50	A	3	1.50	A	3	1.50	
	≥30	All	A	3	1.50	A	3	1.50	A	3	1.50	A	3	2.50	
End suction and split case	≤40	All	C	3	0.75	C	3	0.75	C	3	1.50	C	3	1.50	16
	50 to 125	All	C	3	0.75	C	3	0.75	C	3	1.50	C	3	2.50	10,16
	≥150	All	C	3	0.75	C	3	1.50	C	3	2.50	C	3	3.50	10,16
Cooling Towers	All	Up to 300	A	1	0.25	A	4	3.50	A	4	3.50	A	4	3.50	5,8,18
		301 to 500	A	1	0.25	A	4	2.50	A	4	2.50	A	4	2.50	3,18
		500 and up	A	1	0.25	A	4	0.75	A	4	0.75	A	4	1.50	5,18

Effect of Floor Spans



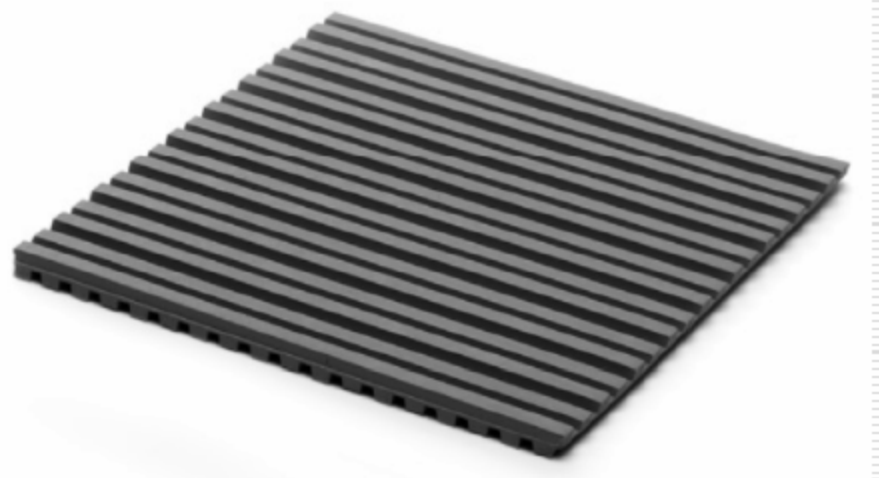
Typical span deflections



- Typical maximum allowable deflection of a floor span is $1/360^{\text{th}}$ of the total span
 - 20' span could result in deflection of .667"
 - Isolator deflection should be at least 10x the actual span deflection
-

Isolator types

Pads – up to 0.15" static deflection
Grade-mount applications



Isolator Types



Elastomer Mounts

"Single Deflection" –
0.25 inches

"Double Deflection" –
0.50 inches

Pad Installation



Open Spring Mounts – Non-Seismic

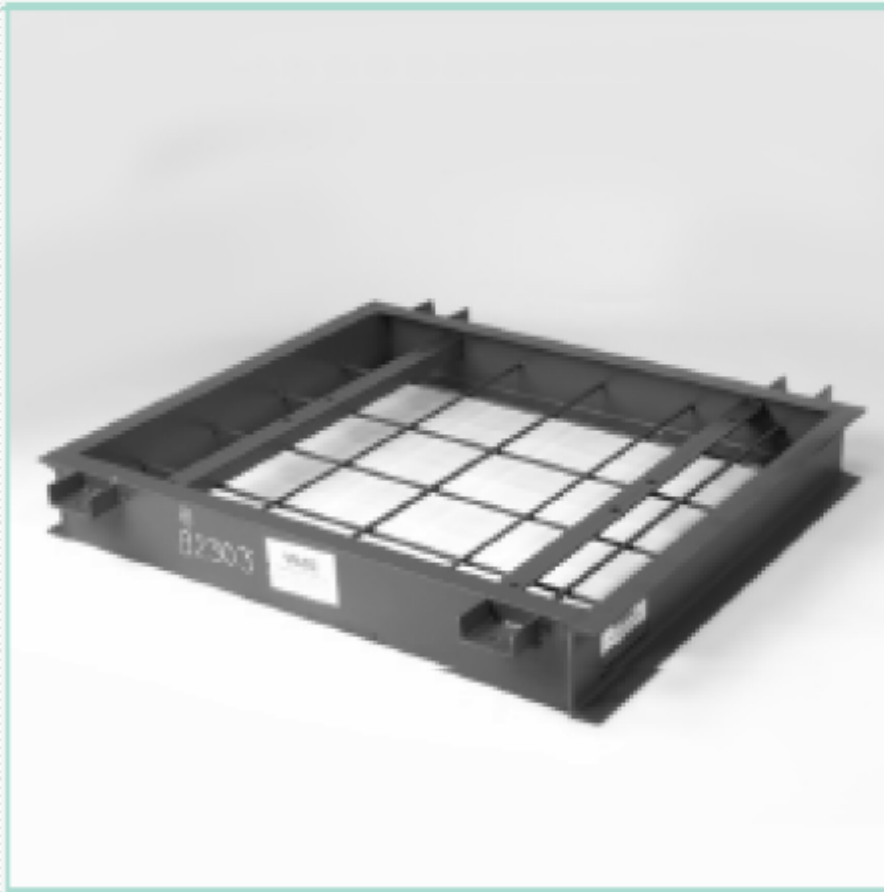


Deflections up to 6"

Open Springs for Inertia Base



Inertia Bases and Steel Bases



Welded steel frames or modular "bolt together" frames

Inertia bases (pumps, compressors, large centrifugal fans) have concrete reinforcement; weighted to reduce movement or lower center of gravity

Steel frames (small utility sets, base-mounted heat pumps) provide a rigid mounting frame and attachment to equipment and isolators

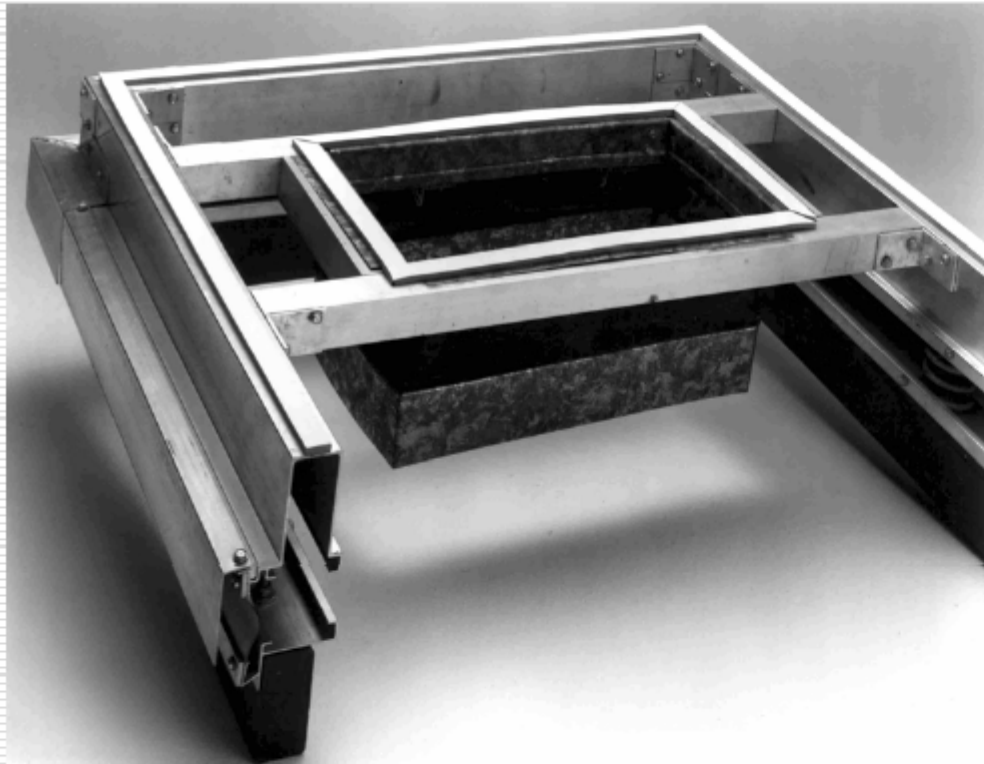
Compressor with Large Unbalanced forces



HSC Pump: T-Shaped Base Supports Suction and Discharge Elbows



Rooftop Isolation Rails



Provide vibration isolation for roof exhaust fans and packaged RTU's

Attach base rail to factory curb

Unit "floats" on open springs within aluminum rails

Up to 3" static deflection

Non-seismic

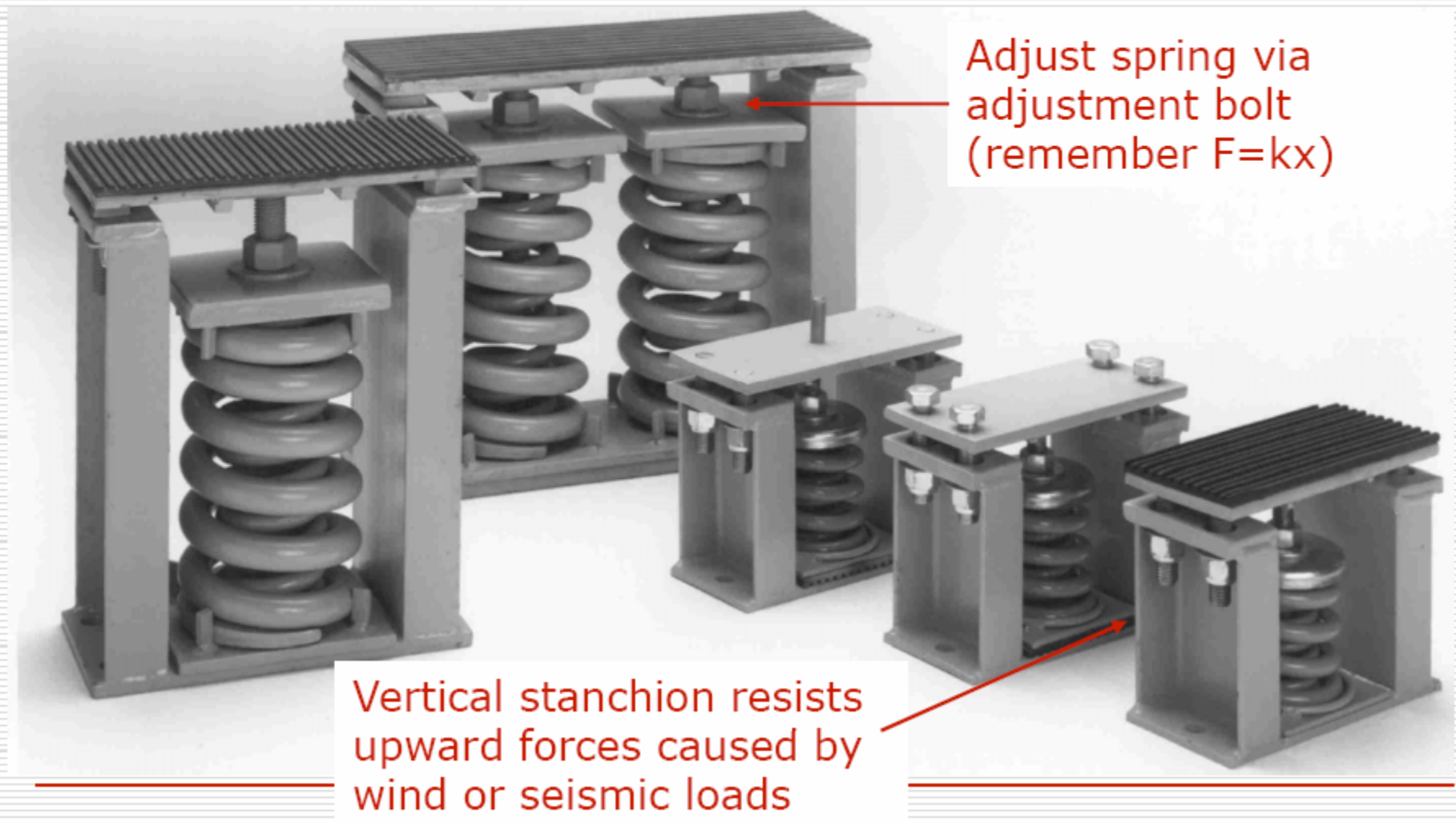
Provide flexible connectors for supply and return duct

Defines "External Isolation"

Restrained Isolators

- ❑ Seismic Loading ~ IBC2003 and IBC2006 requires seismic restraint for nonstructural components (HVAC, plumbing, electrical) in Essential Facilities
 - ❑ “Essential Facilities” includes hospitals, fire, police, air traffic control, communications, and schools designated as emergency shelters
 - ❑ IBC2006 also includes protections from wind events
 - ❑ Wind loading often exceeds seismic loading in this region, especially for tall equipment!
-

Vertically Restrained Isolators



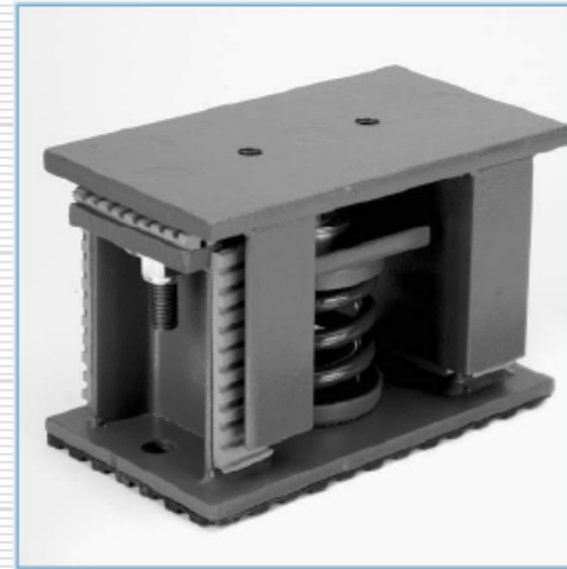
Typical Cooling Tower Isolation



Chiller Isolation



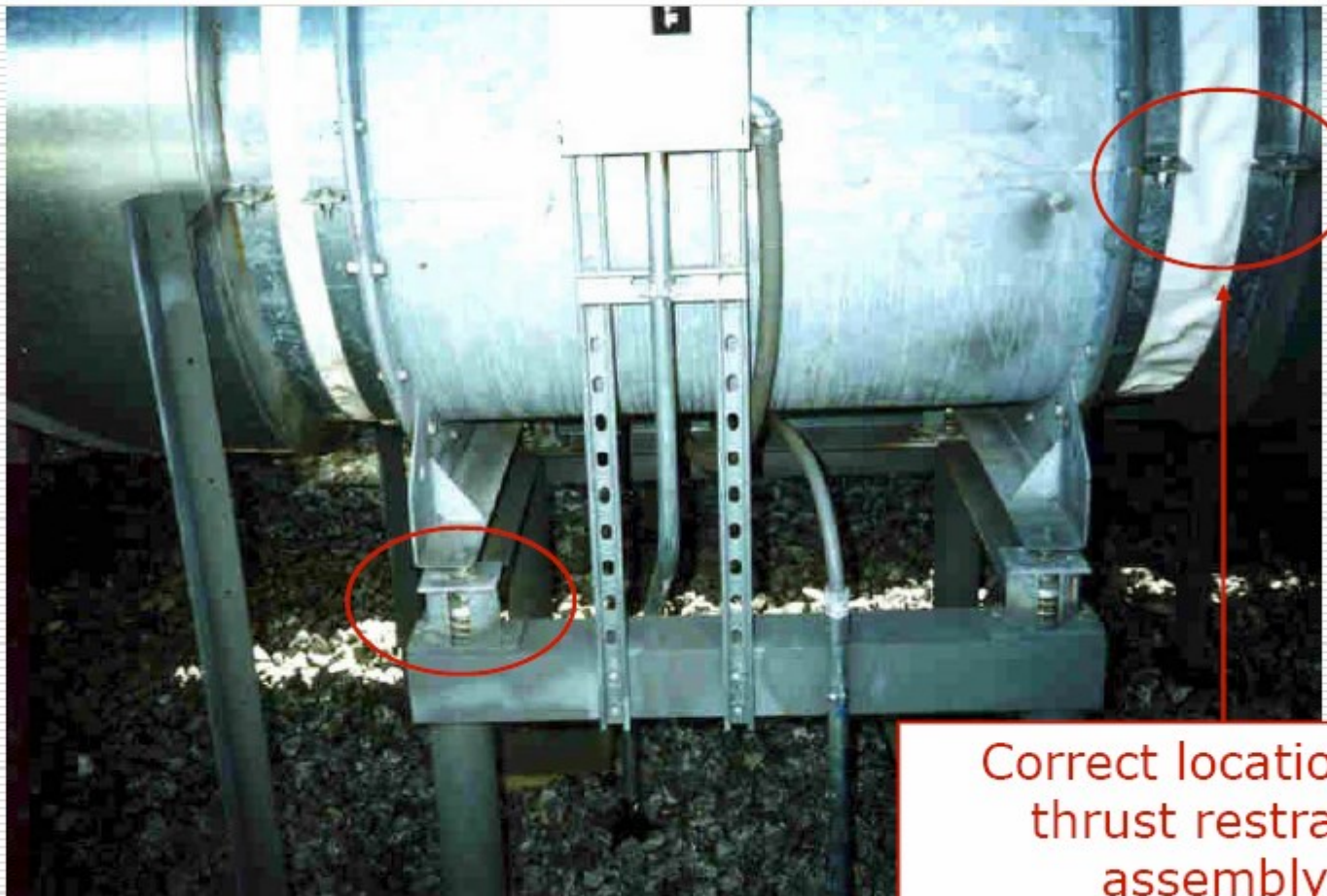
Seismically-Restrained Isolators



Generally consist of open springs with welded steel housings to resist seismic accelerations.

Avoid turning isolated equipment into projectiles.

Vibration Isolation and Seismic Restraint of Inline Fan

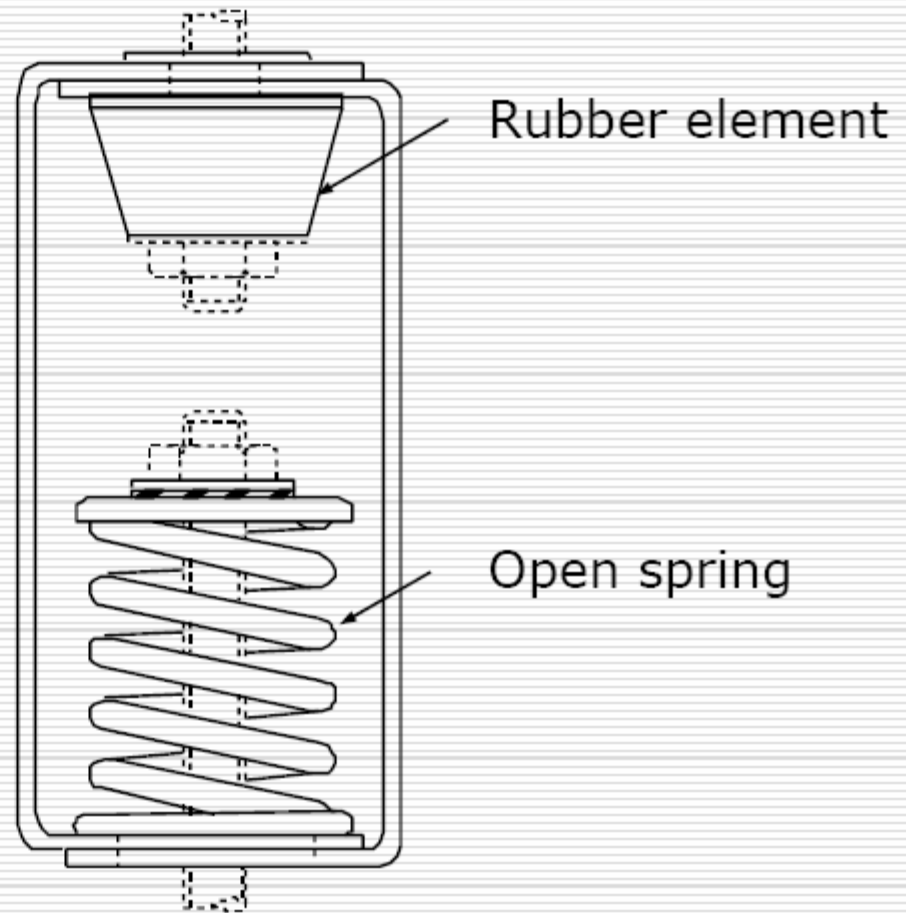


Correct location for
thrust restraint
assembly

Is this installation correct? If not, why?

Isolation Hangers

- Typical for fans, fan coil units, small inline pumps, fan power VAV terminals, suspended duct and pipe
- Deflections typical up to 2-1/2" (larger, too)
- Spring hangers for low frequency
- Rubber element for higher frequencies
- Prevent transmission to deck above



Suspended Fan



Applications - Summary

□ Type of equipment

- Cooling Towers – structural steel and vertically-restrained isolators, rated for seismic loads if required
 - Chillers – pads for grade mount, vertically-restrained isolators for mezzanines and upper floors
 - Base Mount Pumps – pads for grade mount (?), inertia bases with open springs for mezzanines and upper floors; seismic if required
-

Applications Summary Cont'd

- Air Handling Equipment
 - HC/CC: Internally isolated, pads external
 - Packaged DX: Application-dependent; isolation rails or curbs for sensitive spaces
 - Fans: Open spring or spring hanger, steel frame for utility sets
 - Suspended Equipment & Pipe
 - Spring or Spring and Rubber Isolators
 - Prepositioning Isolators for Suspended Pipe
 - Compressors and Vacuum Pumps
 - Inertia Base with high-deflection Isolators
-

Applications and Wrap Up

- ☐ Indoor or outdoor?
 - ☐ Seismic or wind loading?
 - ☐ Varying weight?
 - ☐ Deflection?
 - ☐ Direct mount, frame or base required? (heat pumps, commercial indoor AHU, cooling towers, etc)
-

Conclusion

You should now have a good understanding of:

- The fundamental nature of vibration
- The mechanical parameters involved
- The types of signals encountered
- The relationships between a , v and d
- The units of measurement
- The importance of the measurement chain